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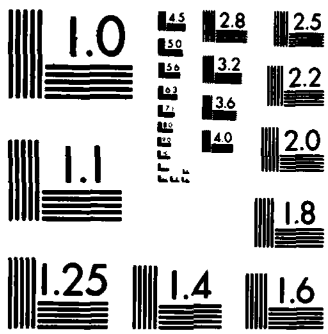

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ADVISORY GROUP FOR AEROSPACE RESEARCH & DEVELOPMENT

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AGARD Report No.719

Three-Dimensional Boundary Layers

NORTH ATLANTIC TREATY ORGANIZATION

DISTRIBUTION AND AVAILABILITY
ON BACK COVER

NORTH ATLANTIC TREATY ORGANIZATION

ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

(ORGANISATION DU TRAITE DE L'ATLANTIQUE NORD)

AGARD Report No.719

THREE-DIMENSIONAL BOUNDARY LAYERS

The information in this report was presented at an AGARD Fluid Dynamics Panel Round Table Discussion on 3D Boundary Layers at the Etat-Major de la Force Aérienne, Brussels, Belgium, on 24 May 1984.

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CONTENTS

INTRODUCTORY REMARKS

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This report contains expanded versions of the oral presentations given at the Round Table Discussions (RTD) on "Three Dimensional (3D) Boundary Layers" held during the Spring Fluid Dynamics Panel meeting in Brussels. The purpose of the RTD was to review the recent progress to guide future panel activities on this subject.

In this introductory chapter an outline of viscous flow equations is first given to define the scope of the RTD. In Figure 1 is shown the hierarchy of equations from the Boltzmann equation on a molecular scale to the macroscopic boundary layer equations. To be noted here are the layers of averaging in the hierarchy and the appearance of formidable complications with each averaging as for example the closure problem with the Reynolds-averaged equations and the inviscid/viscid coupling for the boundary layer limit.

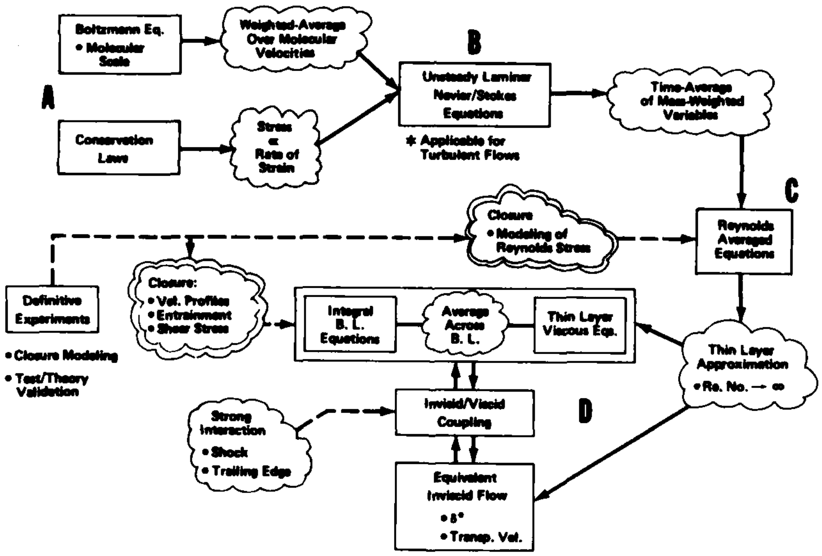

Figure 1. Hierarchy of Viscous Flow Equations

The scope of the RTD was confined to 3D viscous flows in the boundary layer limit (Area D in Figure 1). The primary emphasis was on turbulent flows, particularly on separated flows. In this limit the flow is divided into two parts, the thin viscous layer adjacent to the configuration surface with its downstream wake extension and the outer ambient equivalent inviscid flow incorporating the viscous displacement effects.

Two levels of viscous flow equations in the boundary layer limit were considered, the set of 3D field equations and the corresponding integral boundary layer equations obtained by integrating the field equations across the boundary layer. The latter set of equations with one less independent variable is significantly simpler than the higher-dimensioned field equations.

With the problem divided into two parts, there is then the task of rendering compatible the two solutions; that is, the problem of the inviscid/viscid flow coupling indicated in Figure 1.

In the case of unseparated flow the coupling has been successfully carried out posing the boundary layer problem in the direct mode; that is, inputting the components of the inviscid velocity vector at the surface. The coupling here proceeds in the classical fashion as outlined in Figure 2.

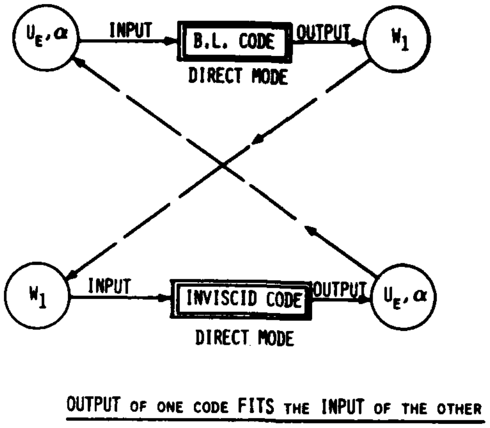

Figure 2. Classical Direct Coupling

In the case of separated flow, few coupled cases have been reported. In Ref. 1 for example, the integral boundary layer equations were coupled to the exact potential code (FL028) and applied to the Boeing 747 swept wing/fuselage at M = 0.86 and 2.7° angle of attack. Following the planar flow experience, in Ref. 1 the integral boundary layer problem was posed in the inverse form where one of the boundary layer variables, the form factor H, and the inviscid flow angle were taken as input functions (see Figure 3). In this case the input/output of the separate problems are no longer compatible as in the direct formulation. An update formula for the H must be additionally postulated.

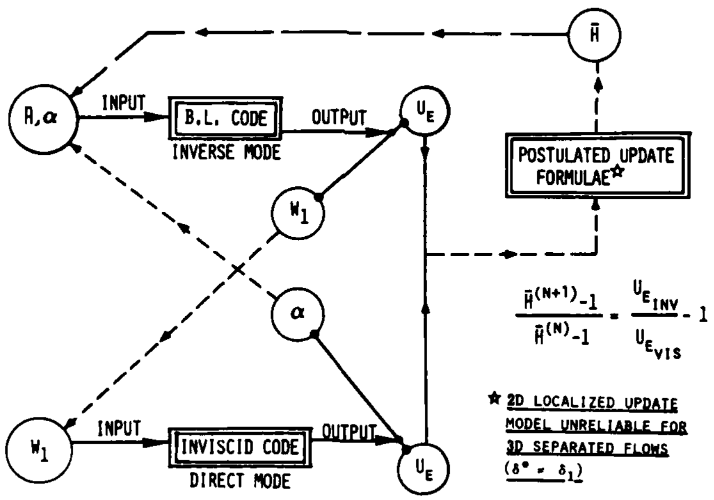

Figure 3. Inverse Coupling

In Reference 1 the simple Carter-type equation from the planar case was found to be adequate for the very mildly separated case at 2.7° angle of attack. In general such simplified planar updating cannot be expected to be applicable for more significant more severely separated cases where the velocity vectors turn spanwise through the boundary layer.

Thus the boundary layer problem must not only be properly posed, but equally important its input/output must mesh compatibly with that of the equivalent inviscid flow to avoid update procedures which are not readily accessible.

In the transonic problem, of major importance is the shock/boundary layer interaction. Although the general experience has been that reasonable results were obtained without special treatment of the interaction, there clearly is a need for a more firmly based treatment of the interaction. In this connection some of the existing asymptotic analyses of the interaction erroneously isolate the boundary layer treating it assuming the outer inviscid flow to be unaffected by the presence of the viscous layer. The same erroneous approach is also made in many "triple-deck" analyses of the st. ng interaction about the trailing edge.

In summary the speakers at the RTD were requested to review the status of the following topics in their respective countries:

1 3D integral or field boundary layer methods,

2. Inviscid/viscid flow coupling,

3. Special topics (separated flow, shock/boundary layer interactions, higher order effects),

4. Data base experiments (existing, ongoing, and planned).

The RTD program was as follows:

1.	Introductory remarks	Dr. H. Yoshihara Boeing Military Airplane Company
2.	France	Professor R. Michel ONERA-CERT
3.	Germany	Professor Dr. H. Hornung DFVLR-Gottingen
4.	Italy	Professor L. Napolitano University of Naples
5.	Netherlands	Dr. B. van den Berg NLR
6.	United Kingdom	Professor A. Young Queen Mary College
7.	United States	Dr. T. Cebeci California State University (Long Beach)

Reference

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THREE-DIMENSIONAL BOUNDARY LAYERS AND SHEAR FLOWS ACTIVITIES AT ONERA/CERT

by

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Among experimental and theoretical research developed at ONERA/CERT, important activities were devoted, these last years, to three-dimensional boundary layers and turbulent shear flows, References 1-20. These activities can be classified into three items :

1) Development, application and control studies of calculation methods for turbulent boundary layers and wakes.

2) Corner flows.

3) Stability and transition in three-dimensional flows.

A review of these studies, complete with typical results, will be briefly presented.

1) Development and control studies of calculation methods for turbulent boundary layers and wakes

Integral methods based on global boundary layer equations as well as field methods solving local equations, have been developed; their principles and hypothesis are given in Figures 1 and 2.

Integral methods use closure relationships given from similarity solutions which are established from a classic mixing length scheme.

On the other hand, field methods need turbulence models which may vary from a very simple mixing length scheme to a more complicated 4-equations model for the turbulent shear stresses.

Systematic applications of these calculation methods lead, in several cases, to satisfactory results, in good agreement with experimental values. However, some difficult cases have been encountered; for instance, the experiment performed at NLR on the boundary layer of a swept wing having a three-dimensional separation line, in the rear part of the wing, Reference 21.

As shown in Figure 3, none of the calculation methods tried were in good agreement with the results of this experiment. We had to discuss the problem of the direction of the shear-stress vector, by testing the effect of the non-alignment parameter $T = \tan \delta_\tau / \tan \delta_g$

The influence of that parameter is effective, but does not allow, nevertheless, to get results close to the experimental values.

On the other hand, it is interesting to notice that an inverse mode calculation, avoiding singularity problems close to the separation line, gives, more easily, coherent results with the experiment, Figure 4.

An experimental study of the wake of a swept wing has been extensively developed at the Aerothermodynamics Department (ONERA/CERT). It provides us with detailed results for streamwise and crossflow mean velocity profiles, Figure 5, as well as for components of the turbulent shear stress tensor, Figure 6. Both integral and field methods have been applied to this case with results in good agreement with experimental values as shown in Figure 7.

2) Corner flows

Flows along corners formed by two intersecting walls exist in a great number of engineering applications, such as wing-body junctions or non-circular diffusers. In all of these situations, a transverse circulatory motion (secondary flow) develops and greatly alters the main flow. In fact, there are two mechanisms for the generation of secondary flows: one is due to Reynolds stress gradients, the other is induced by lateral skewing of a shear layer.

A theoretical study has been developed, in order to establish corner flow calculation methods. The governing equations utilized and the corresponding assumptions are given in Figure 8.

The main problem is to obtain a satisfactory turbulence modeling. Special care must be paid to the Reynolds stresses v'^2, w'^2 and $v'w'$, which are directly responsible for the generation of secondary flows. Models used in recent calculations are given in Figure 9. These methods have been applied to internal flows (straight or curved ducts of rectangular cross section) as well as to external, unbounded corner flows, such as those which exist in the vicinity of a wing-fuselage junction. Two examples of such computations are given in Figure 10. The first one (MOJOLA and YOUNG experiments, Ref. 22) deals with a straight corner flow, in which the secondary flow takes the form of streamwise vortices, closely related with the turbulence anisotropy (second kind secondary flow). The second example (SHABAKA's experiments Ref. 23) concerns a simplified wing-body junction: the computation describes the evolution of a first kind secondary flow, induced by the three-dimensional separation occuring upstream of the wing leading edge.

An experimental study of a corner flow was also undertaken. It is concerned with the flow development in the vicinity of the junction between a high sweep, high incidence wing and the wind tunnel floor. The pressure field, the mean velocity components and the six elements of the Reynolds stress tensor have been measured. They will make it possible to check in a detailed way the accuracy of the calculation methods, in particular the validity of the turbulence models. The wing model in the tunnel and pressure and secondary velocity measurements are shown in Figure 11.

3) Boundary layer transition in three-dimensional flows

This problem has been studied in detail when transition occurs on swept wings. Two different types of transition may exist:

- at low Reynolds numbers: transition is due to streamwise instability; it appears in the positive pressure gradient region, after the maximum of the external velocity distribution;

- at higher Reynolds numbers: transition moves forward, towards the leading edge, in the negative pressure gradient region and is due to crossflow instability.

Based on theoretical results given by the laminar stability theory and on experimental results obtained at the transition point, criteria have been established as regards to either streamwise or cross-flow instabilities and are shown in Figures 12-14. By applying them, transition onset can be predicted. Then, an intermittency method is considered for calculating the boundary layer within the transition region; so, the whole boundary layer can be computed towards the trailing edge, where its momentum thickness can be related to the drag coefficient. An example of results shown in Figure 15 clearly shows the non-stabilizing effect of the sweep angle with an increase in the drag coefficient.

An important phenomemon, displayed with wall flow visualization, must be underlined. It reveals a closely spaced streak pattern nearly aligned with the external streamlines, Figure 16. These streaks exhibit a periodical spanwise evolution of the skin-friction coefficient.

A detailed experimental study of the boundary layer, subject to these streaks, has been undertaken. It shows an important evolution in the spanwise direction of the mean velocity profiles and of the perturbation profiles measured within the laminar boundary layer, Figure 17.

The laminar stability theory permits verification of the existence of these streaks, closely related to the amplification of zero-frequency waves. Stability calculations provide us with results in good agreement with experimental values as regards either to the wavelength or the amplitude of such a phenomenon.

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	BASIC EQUATIONS	CLOSURE RELATIONSHIPS
THREE-DIMENSIONAL TURBULENT BOUNDARY LAYERS	- x GLOBAL MOMENTUM EQUATION - z GLOBAL MOMENTUM EQUATION - ENTRAINMENT EQUATION	SIMILARITY SOLUTIONS USING A MIXING LENGTH MODEL → STREAMWISE AND CROSSWISE VELOCITY PROFILES → RELATIONSHIPS BETWEEN CHARACTERISTIC BOUNDARY LAYER THICKNESSES AND SKIN FRICTION COMPONENTS
SWEPT WING WAKES	- x MOMENTUM EQUATION - z MOMENTUM EQUATION - ENTRAINMENT EQUATION	- STREAMWISE PROFILE : $\frac{U_e - U}{U_c - U} = \left[\left(\frac{y}{\delta}\right)^{3/2} - 1\right]^2$ - CROSSFLOW PROFILE : SIMPLIFIED (TRIANGULAR) REPRESENTATION IN POLAR PLOT W(U)

INVERSE MODE	SAME EQUATIONS SAME CLOSURE	DISPLACEMENT THICKNESSES δ_1 AND δ_2 PRESCRIBED EXTERNAL VELOCITY DISTRIBUTIONS AND OTHER BOUNDARY LAYER CHARACTERISTICS CALCULATED

Figure 1. Calculation Methods For Turbulent Boundary Layers And Wakes- Integral Methods

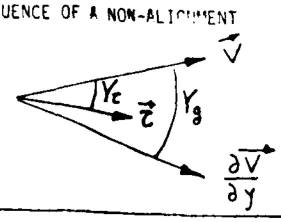
THREE-DIMENSIONAL BOUNDARY LAYERS SWEPT WING WAKES	- LOCAL BOUNDARY LAYER EQUATIONS + - TRANSPORT EQUATIONS FOR TURBULENT QUANTITIES - MIXING LENGTH MODEL - 2 EQUATIONS MODEL : K, ε - 4 EQUATIONS MODEL : K, ε, $u'v'$, $w'v'$ - SPECIFIC PROBLEM : - DIRECTION OF THE TURBULENT SHEAR-STRESS VECTOR ? - TESTS ON THE INFLUENCE OF A NON-ALIGNMENT PARAMETER : $T = \frac{\tan \gamma_\tau}{\tan \gamma_g}$ 

Figure 2. Calculation Methods For Turbulent Boundary Layers And Wakes - Field Methods

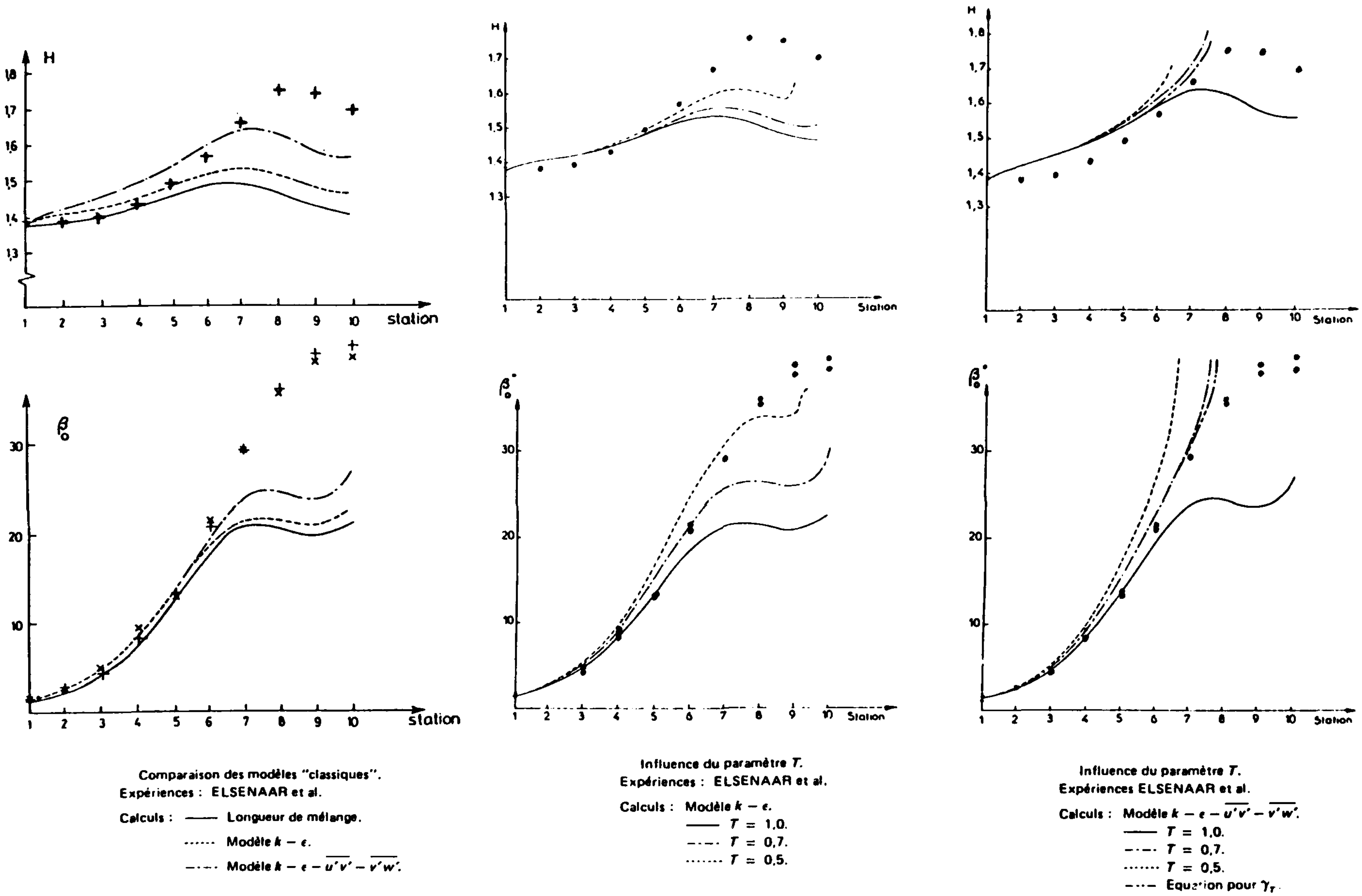


Figure 3. Comparison Of Calculation Methods With Experiment (Ref. 21)

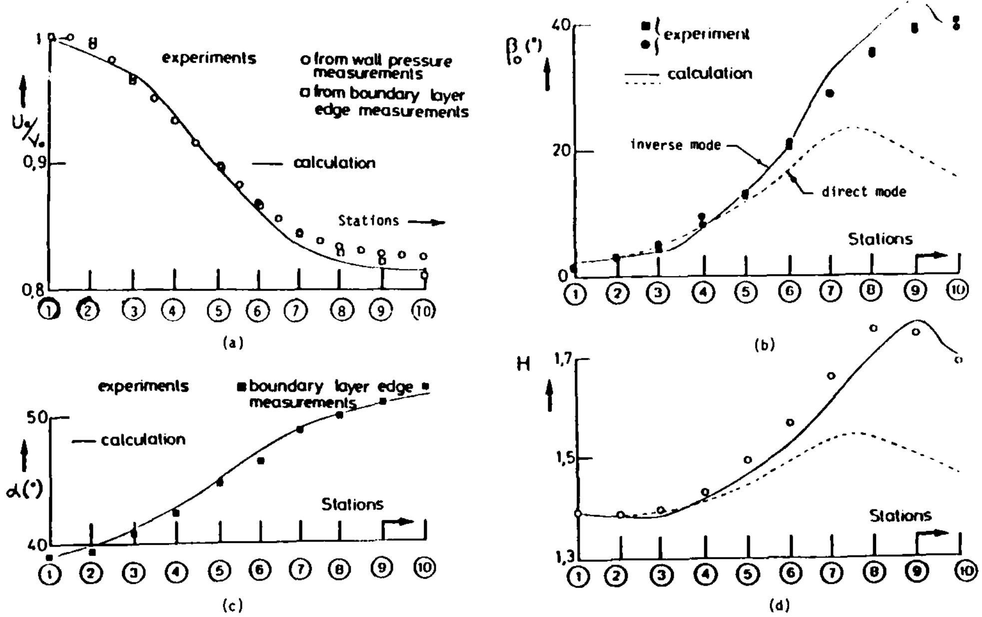


Figure 4. Comparisons Of Calculations With An Integral Method In Inverse And Direct Mode; (a) External Velocity, (b) External Flow Angle, (c) Limiting Wall Streamline Angle, (d) Streamwise Shape Parameter

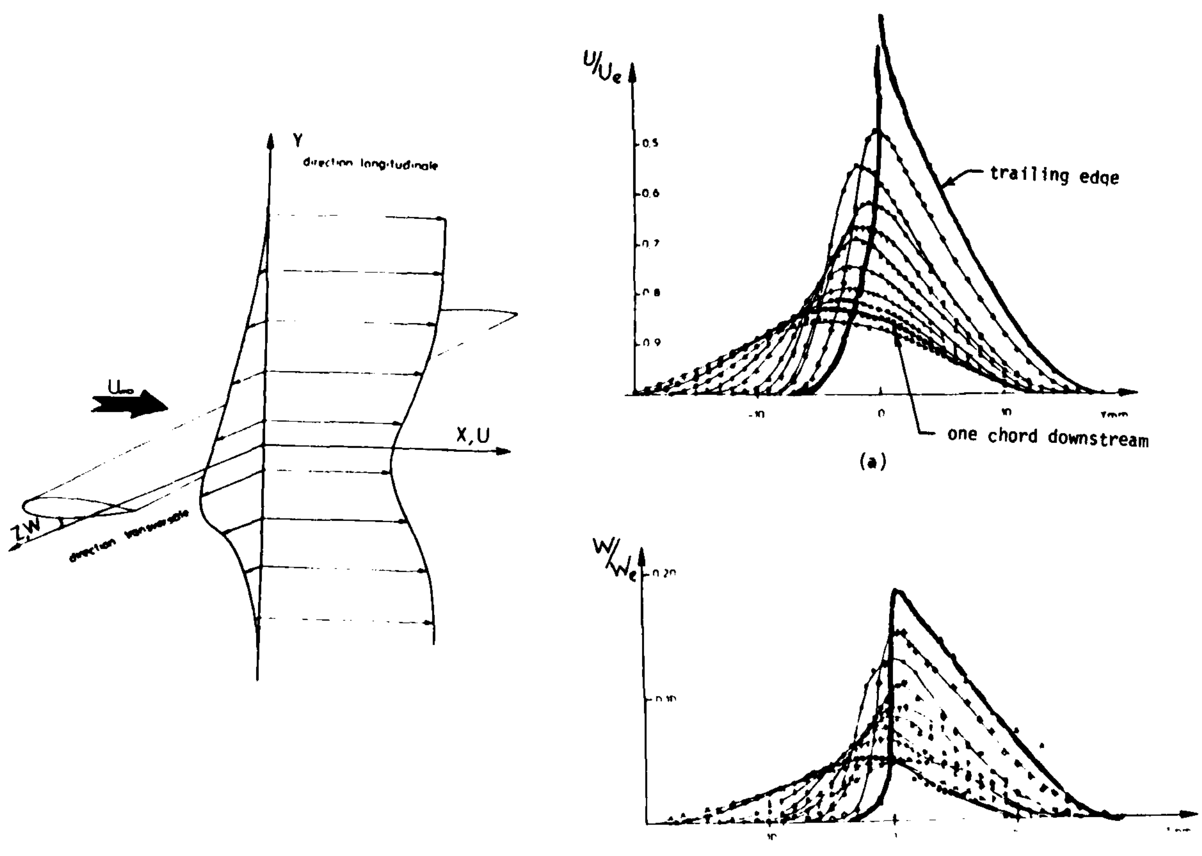


Figure 5. Measurements In The Wake Of Swept Wing (Velocity Profiles (a) Longitudinal, (b) Transverse)

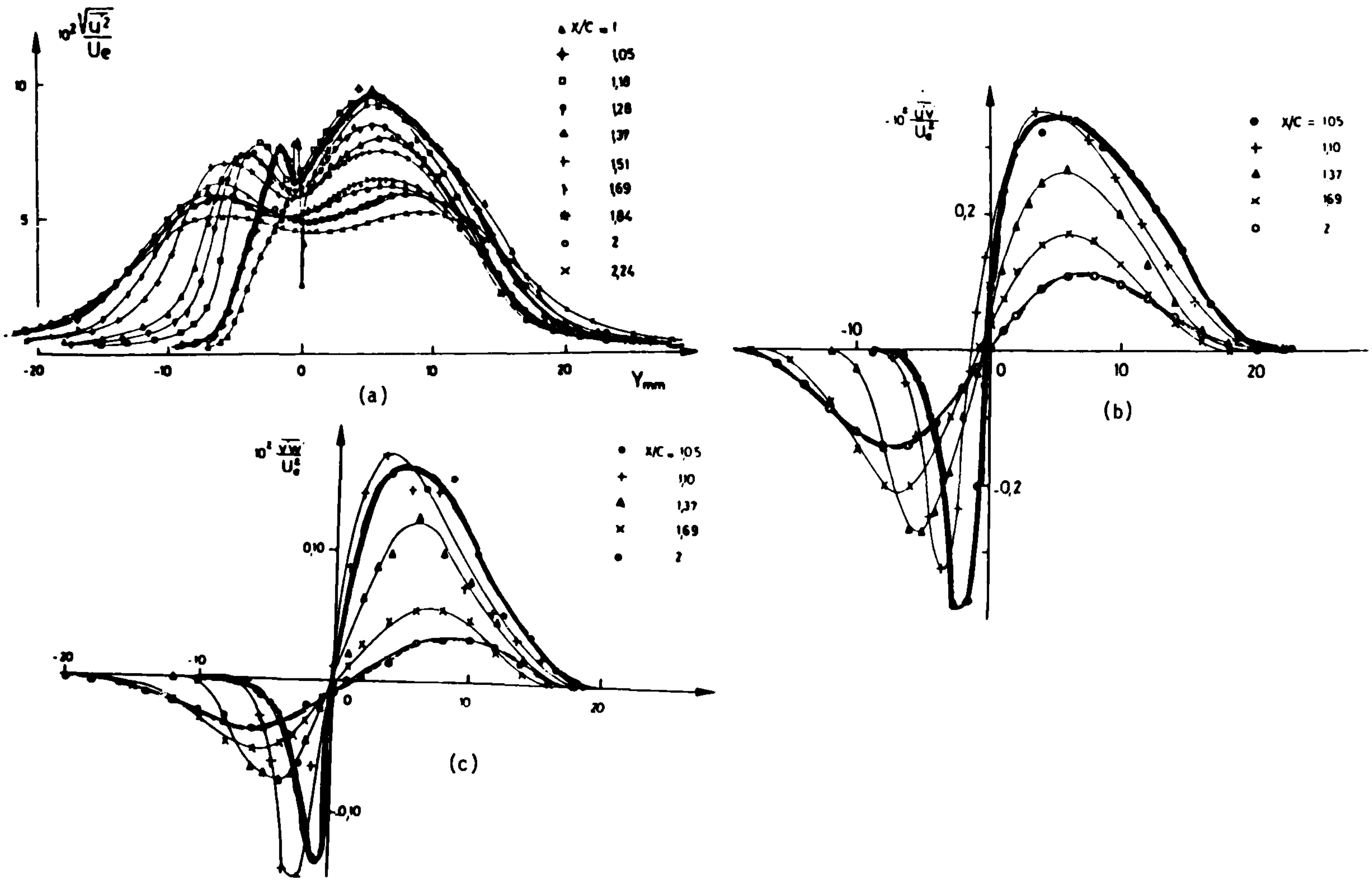


Figure 6. Measurements In The Wake Of Swept Wing; (a) Turbulence Intensity, (b),(c) Shear Stress

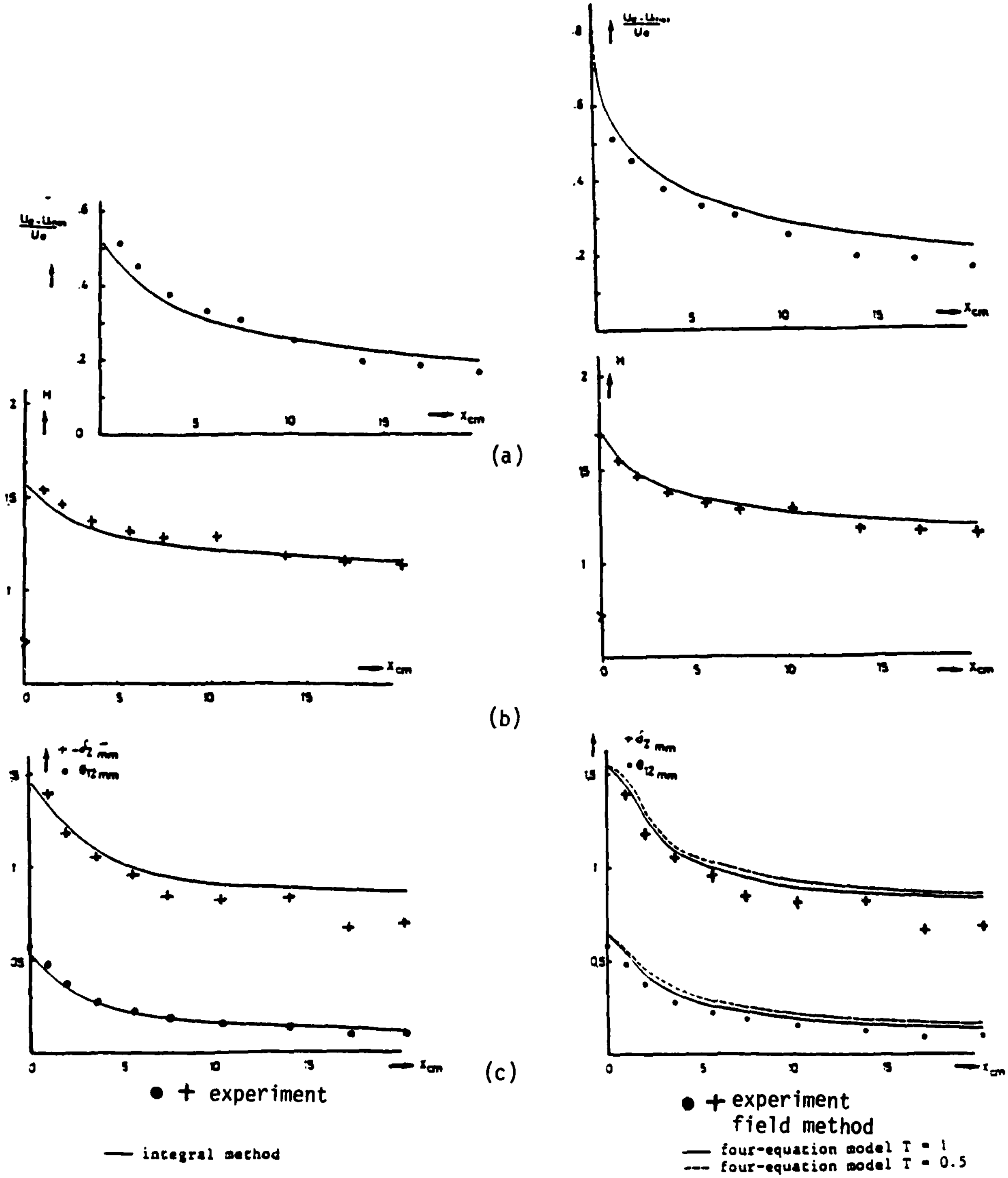


Figure 7. Swept Wing Wake; Comparison Of Integral And Field Calculation Methods With Experiments; (a) Minimal Velocity, (b) Shape Parameter, (c) Transversal Characteristic Thickness

THEORETICAL STUDIES

CALCULATION METHOD SOLVING :

- THE STREAMWISE MOMENTUM EQUATION
- THE EQUATION FOR THE STREAMWISE VORTICITY COMPONENT ω
- TWO POISSON'S EQUATIONS FOR THE SECONDARY VELOCITIES V, W

SYSTEM PARABOLISED IN THE X-DIRECTION
(X-DIFFUSION TERMS AND REYNOLDS STRESSES DERIVATIVES NEGLECTED)

TWO KINDS OF VORTICES ENCOUNTERED :

- FIRST KIND VORTICES, DUE TO STREAMLINES CURVATURE
- SECOND KIND VORTICES, DUE TO THE TURBULENCE ANISOTROPY

Figure 8. Corner Flows: Principle Of Calculation Methods

STRESSES APPEARING IN THE X MOMENTUM EQUATION

$$\boxed{U'V' = \nu_t \frac{\partial U}{\partial y}} \qquad \boxed{U'W' = \nu_t \frac{\partial U}{\partial z}}$$

ν_t : - MIXING LENGTH MODEL OR
- K, ϵ TRANSPORT EQUATIONS

STRESSES APPEARING IN THE ω EQUATION

- EDDY VISCOSITY MODEL

$$V'^2 - W'^2 = -2\,\nu_t\left(\frac{\partial V}{\partial y} - \frac{\partial W}{\partial z}\right)$$

$$V'W' = -\,\nu_t\left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y}\right)$$

- ALGEBRIC STRESS MODEL (SIMPLIFIED TRANSPORT EQUATIONS FOR U', V', W', U'V', U'W', V'W', CONVECTION AND DIFFUSION NEGLECTED)

A) SMALL CROSS FLOW

$U >> V.W$

$$V'^2 - W'^2 = \frac{4}{C_1}\,\frac{4\,C_2 - 1}{11}\,\frac{K}{\epsilon}\left(U'V'\,\frac{\partial U}{\partial y} - U'W'\,\frac{\partial U}{\partial z}\right)$$

$$V'W' = \frac{2}{C_1}\,\frac{4\,C_2 - 1}{11}\,\frac{K}{\epsilon}\left(U'V'\,\frac{\partial U}{\partial z} + U'W'\,\frac{\partial U}{\partial y}\right)$$

B) LARGE CROSS FLOW : ALGEBRIC SYSTEM WITH 6 EQUATIONS FOR THE 6 UNKNOWNS U', V', W', U'V', U'W', V'W'

Figure 9. Corner Flows: Closure Relationships, Turbulence Models

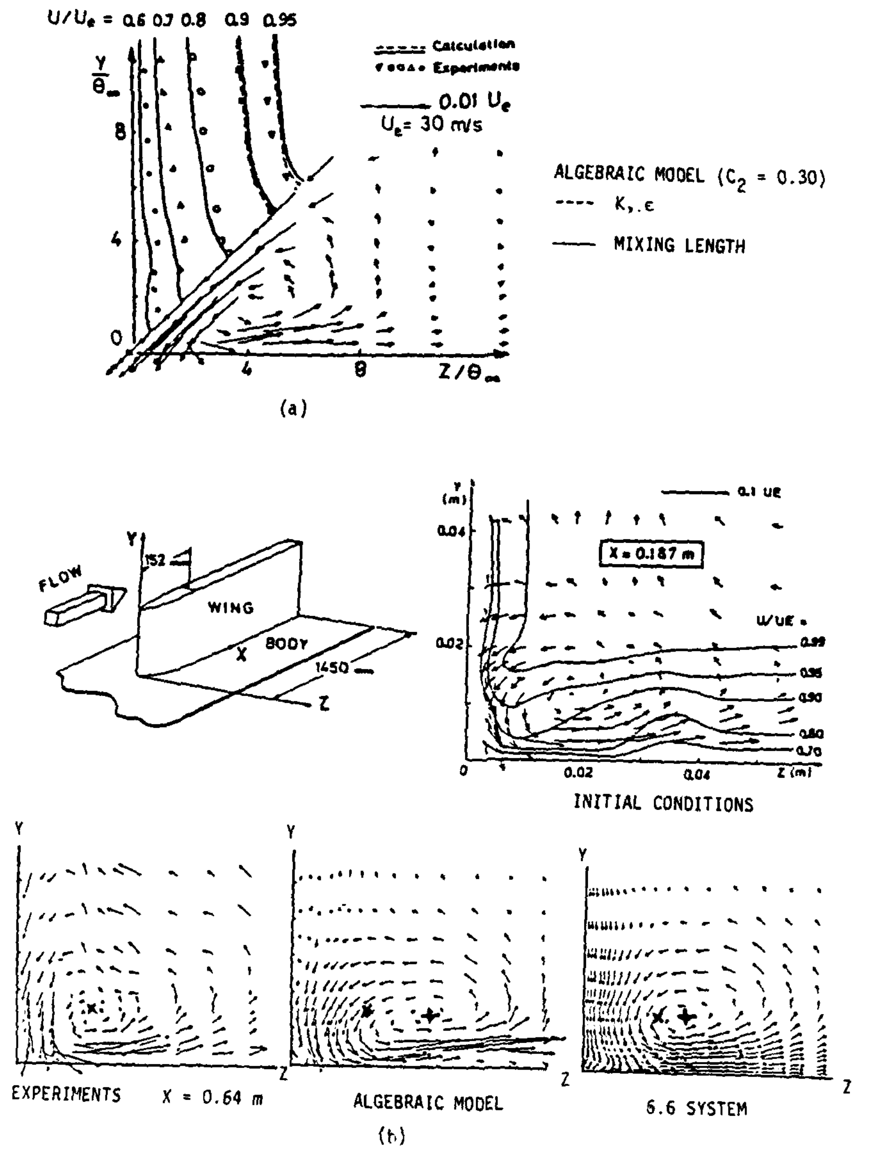


Figure 10. Corner Flow, Comparison Of Theory And Experiment; (a) Second Kind Secondary Flow (Experiments Of Mojola And Young, Ref. 22), (b) First Kind Secondary Flow (Experiments Of Shabaka, Ref. 23)

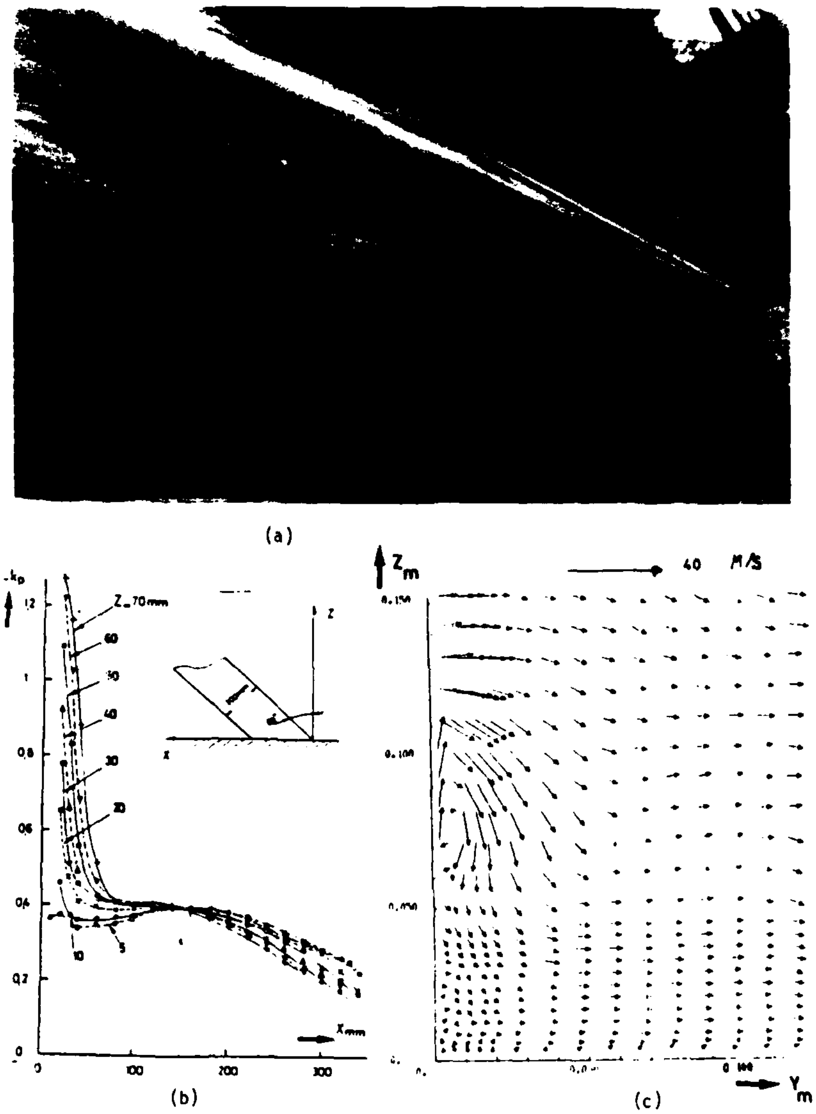


Figure 11. Experimental Study Of The Flow In The Vicinity Of A Swept Wing-Fuselage Junction: (a) Wall Flow Visualization, (b) Pressure Field, (c) Secondary Flow

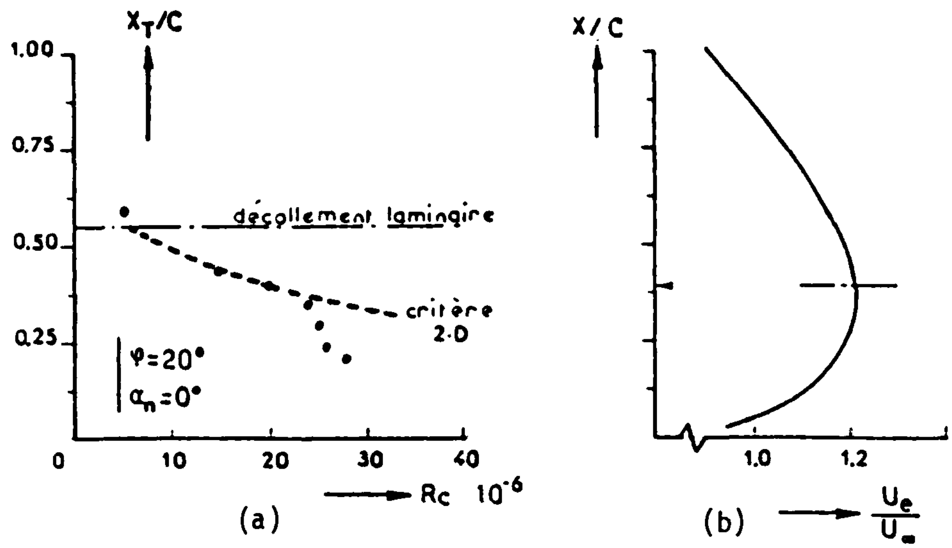


Figure 12. Transition Location VS Reynolds Number On A Swept Wing; (a) Streamwise Instability At Low Re Number, (b) Cross-Flow Instability At Higher Re Number

-Established in 2-D flows from :

- **-stability calculations for similarity profiles**
- **-MACK's relation n=f(Tu)**

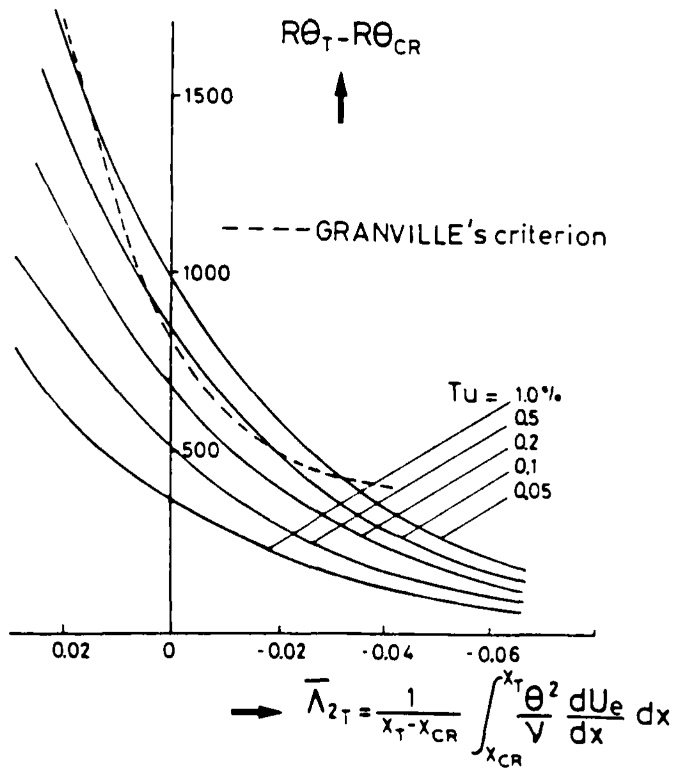


Figure 13. Criterion For Transition Due To StreamWise Instability

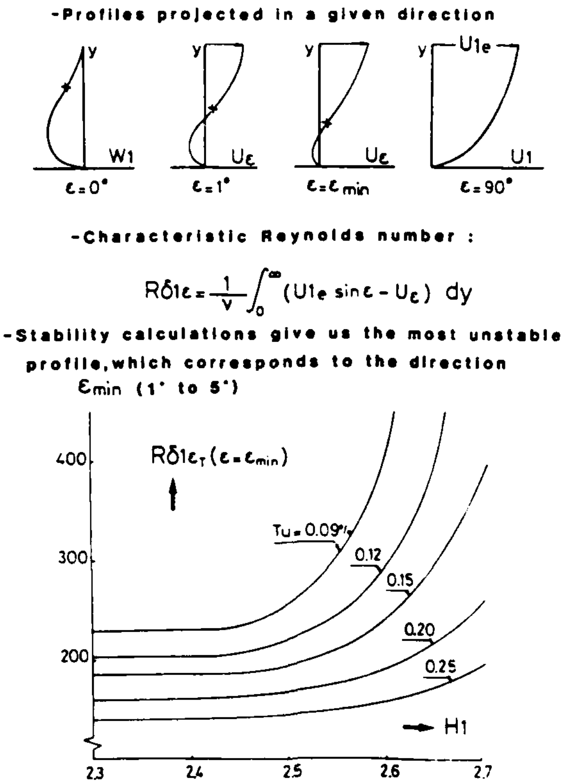


Figure 14. Criterion For Transition Due To Cross-Flow Instability

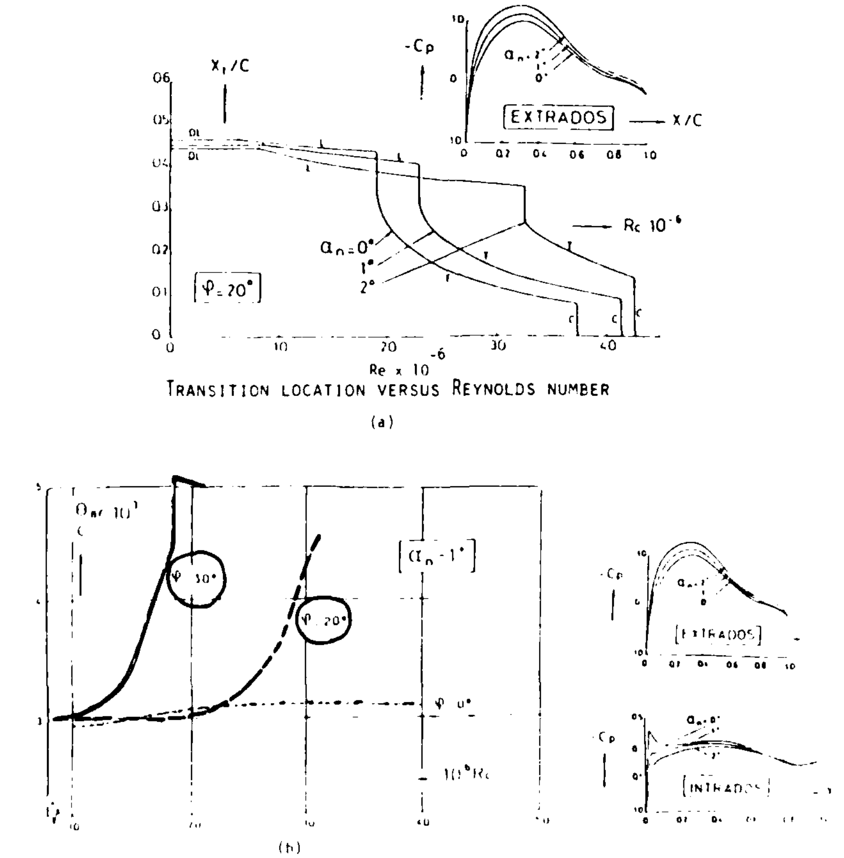


Figure 15. Results Of Application Of Transition Criterion And B.L. Calculations On A Swept Profile (ONERA OAP 01); (a) Transition Location VS Re Number, (b) Momentum Thickness At Trailing Edge VS Re Number

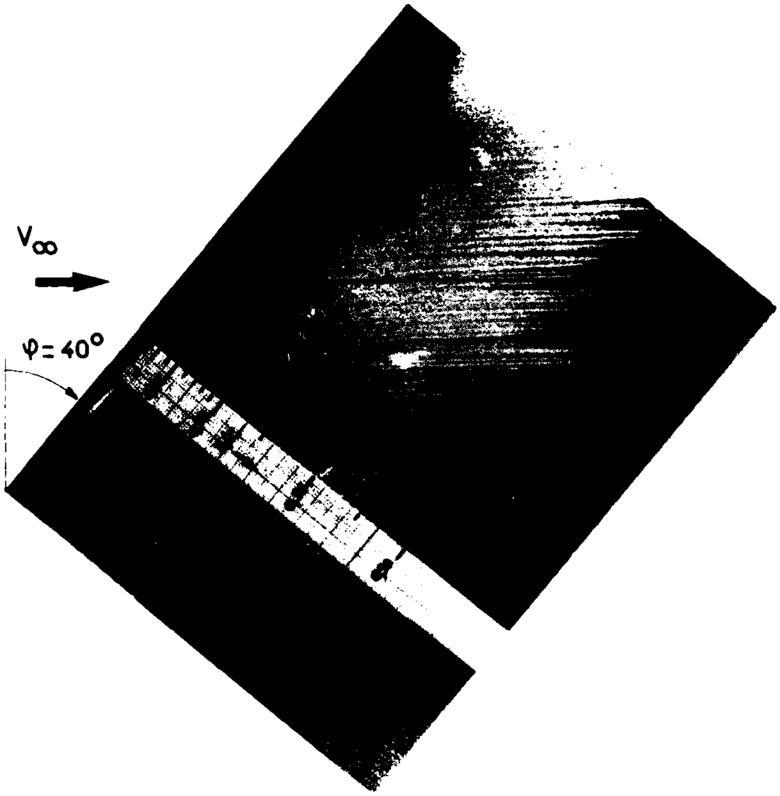


Figure 16. Wall Visualization On A 40 Sweep Angle Wing; Streaks Pattern Exhibiting A Spanwise Evolution Of The Skin Friction Coefficient

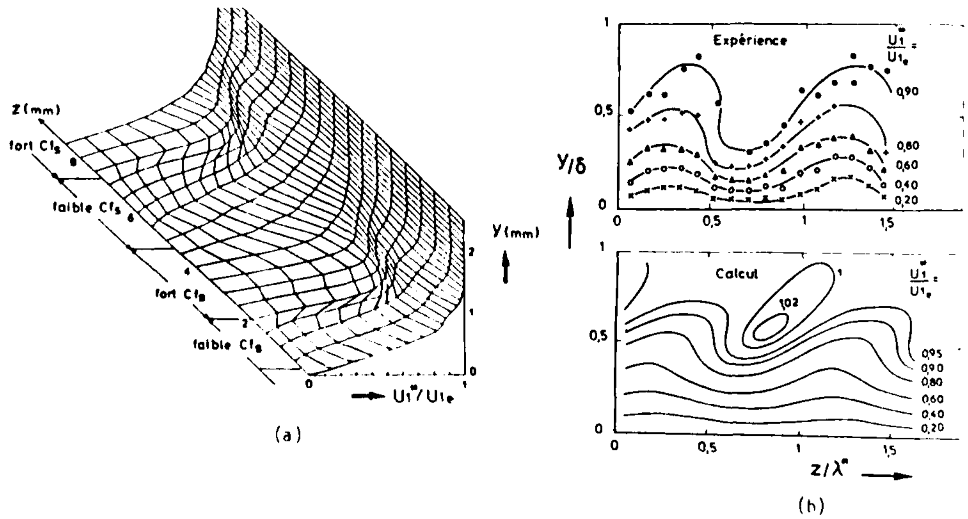


Figure 17. Comparison Between Experiment And Stability Theory Analysis Of 40 Swept Wing Streak Patterns; (a) Measured Evolution Of The Laminar Velocity Profile Across The Streaks, (b) Comparison Of Isovelocity Lines Between Experiment And Laminar Stability Calculations Showing Amplification Of Zero Frequency Waves

THREE-DIMENSIONAL BOUNDARY LAYERS — A REPORT ON WORK IN GERMANY

by

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Summary

The work of selected industrial, university and research establishment groups on three-dimensional boundary layers is collected and presented briefly. The information is far too estensive and heterogeneous to go into detail, and the report should only be regarded as a guide to the quoted references in which the details may be found by interested readers.

1. The problem

The step from two-dimensional to three-dimensional boundary conditions introduces the following new features and difficulties:

- The origin of mean streamlines arriving at a point on a body depends on the distances from the wall, with the consequence that marching techniques appropriate to computations in two-dimensions must be modified or they fail in three-dimensions.
- Transverse pressure gradients generate streamwise vorticity at the wall.
- Transition predictions are inadequate.
- Turbulence models of two-dimensional flows are inadequate for three-dimensional applications.

These difficulties arise even in situations with smooth boundary conditions. In "complex" turbulent flows or in flows with "separation" the complexity of the interaction with the inviscid outer flow is also much greater than in the two-dimensional case.

2. The approaches

The various groups contacted (see 4.) have approached the problem in different ways including the following:

- Numerical techniques

  Integral methods
  Direct methods
  Inverse methods
  Coordinate optimization
  Numerical experiments with turbulence models

- Experimental techniques

  Hot wire anemometry and variations (e.g. pulsed wire)
  Doppler anemometry
  Preston tubes, hot films
  Mean profiles
  Pressure-velocity correlation

- Data base experiments

  Two-dimensional upstream with downstream three-dimensional pressure field
  Ship stern
  Ellipsoid
  Airbus A310

3. Previous coordination of European work

In many cases one approach is not enough to obtain a complete picture. The recognition of this fact has brought about the following previous European coordinations:

- Trondheim Trails 1980, Ref. 44
- Ship boundary layers, Ref. 1, 2
- IUTAM Symposium Berlin 1982, Ref. 3
- Eurovisc activities, Ref. 4
- Euroexpt., Ref. 5

Particular importance should be attached to completeness of information on any experiment. For a check list on this topic we recommend Bertelrud's list, Ref. 5.

4. The groups contacted in Germany

Contributions have been provided for this summary by the following groups through the individuals listed in parentheses:

- Industry

Dornier	(Stock)
MBB-UF, Munchen	(Hirschel)
MBB-UT, Bremen	(Elsholz)

- Universitities

Erlangen	(Durst)
Karlsruhe	(Felsch)
Berlin	(Fernholz)
Aachen	(Krause)
Hamburg	(Kux)
Karlsruhe	(Rodi)

- Establishments

DFVLR, SM-TS	(Kordulla)
DFVLR, SM-ES	(Meier)

Brief summaries of the relevant research in the groups is given in Sections 5-15

5. Dornier GmbH Friedrichshafen (H.W. Stock)

5.1. Axisymmetric and three-dimensional boundary layer computation

An integral method is used which incorporated the following features:

- Non-orthogonal curvilinear coordinates (not streamline coordinates).
- Howarth-Stewartson transformation for compressibility.
- Two-parameter Coles velocity profile for streamwise profiles.
- One-parameter Mager profiles for cross-flow direction.
- Wall shear is calculated by differentiating Coles profiles with respect to streamwise distance.
- Turbulence model incorporates lag-entrainment method.
- Metric coefficients are calculated either analytically or numerically in Cartesian coordinates using bicubic splines.
- Integration by explicit intermediate step method in x-direction. Hyperbolic nature of differential equation (domains of dependence and of influence) is taken into account in computing the y-dervatives. Central, backward or forward differences are chosen according to the local situation.
- Initial conditions, stagnation points or lines are calculated analytically.
- Symmetry in the boundary conditions is imposed on the solution.

This scheme has been applied to a number of three-dimensional, quasi-two-dimensional and axisymmetric flows, e.g. Ref. 6-13.

An example of the application of these methods is shown in Fig. 5.1 in comparison with other computations of the same experiment. The experiment is that of van den Berg and Elsenaar, which was a three-dimensional boundary layer on a flat plate, generated by a skew pressure field due to a displacement body.

5.2. Viscous-Inviscid Coupling

Work is progressing on extending the existing two-dimensional codes, full potential plus boundary layer, Ref. 14, and Euler plus boundary layer, Ref. 15, to three dimensions. The trailing edge condition is modelled correctly by the Euler method, but not by the potential.

5.3. Navier-Stokes Solutions

Considerable succeess has been achieved in two-dimensional problems, including shock-boundary layer interaction and cryogenic real gas effects. Work on the extension of these techniques to three-dimensions is progressing.

6. Messerschmidt-Bolkow-Blohm GmbH, Military Aircarft Division (E.H. Hirschel)

6.1. Work on three-dimensional boundary layer problems.

- Data pre-processing for boundary-layer computations.

  Representation of general wing and fuselage geometries, construction of boundary-layer coordinates, transformation of inviscid external flow-field data, coordinated for higher-order boundary layer considerations.

- Integration of boundary-layer equations.

  Development of finite-difference solutions for special applications, expecially for laminar boundary-layer control (oblique and normal suction and blowing, heating and cooling). Production runs for general boundary-layer problems are made with the integral methods of Cousteix (CERT/DERAT Toulouse).

- Post-processing of results of boundary-layer computations.

  Evaluation of boundary layer results (boundary-layer displacement thickness, equivalent inviscid source distribution, skin-friction lines, integral friction forces) for interpretatuon and for coupling procedures.

- Integration of boundary layer results with regard to separation.

  Study of separation indicators (convergence of skin-friction lines. bulging of boundary-layer thickness and displacement thickness contours, occurence of $|\tau|$-minimum lines, applications of topological rules.

- Application of these boundary-layer techniques to wing and fuselage (airplanes, missiles, car bodies) problems.

- Passive boundary-layer control by configuration shaping (afterbodies).

- Passive and active (suction) laminarization of boundary layers on swept wings, 3-D hydrodynamic stability and transition criteria.

- Euler simulation of separated flows.

  Principles of vorticity generation, coupling of boundary-layer solution with Euler solution at separation lines.

- Local solutions of Navier Stokes equations.

  Coupling of inviscid methods with boundary-layer method and local Navier-Stokes solutions in regions where the boundary layer leaves the surface (wing trailing-edge problems, base-flow problems, separation in general).

An example of computational results is shown in Figure 6.1. Detailed information on all all of the topics may be found in Ref. 16-20.

7. Messerschmidt-Bolkow-Blohm GmbH-UT (E. Elsholz)

7.1 Boundary-layer methods

Here also the integral method of Cousteix, Ref. 20, together with Hirschel's metric routines, Ref. 17, are used, Ref. 21. Good results are obtained on fuselages with the outer flow calculated by a potential method, see Fig. 7.1. Work is progressing on implementing Schwamborn's, Ref. 22, three-dimensional finite-difference method for laminar leading edge flows on swept wings.

7.2 Viscous-Inviscid Coupling

The approach here is to use iterative coupling of Cousteix's integral method with a three-dimensional panel method, Ref. 23, 24, via the surface transpiration concept. Though stable results may be obtained for fully attached flows over three-dimensional clean wings, Ref. 25 and Fig. 7.2, trailing edge effects are not yet included, which causes the lift to be overpredicted.

7.3 Experiments

Experimental investigation of the A310 model in the DNW will incorporate pitot rake measurements of the boundary-layer at the wing fuselage fairing (upper side).

8. University of Erlangen Lehrstuhl fur Stromungsmechanik (F. Durst)

Though three-dimensional boundary-layers are not part of the research program of this department, the experimental techniques that are being developed, in hot-wire and laser-doppler anenometry are directly applicable and deserve a brief mention. Information may be found in Ref. 26, 27.

9. Technical University of Berlin ,Hermann-Fottinger-Institut (H. H. Fernholz)

The group under Professor Fernholz was active in the field of experimental three-dimensional turbulent boundary-layer research until recently and he is taking an active part in the planning of the work under the Euroexpt. program.

The approach of this university group as distinct from the aims of the industrial groups is not to develop computational techniques for predicting practical flow situations but rather to study a particular flow situation (axially symmetric boundary-layer encountering an asymmetric pressure distribution) experimentally in considerable detail in order to gain a better understanding of some of the funamental features of three-dimensional turbulent boundary-layers. The results include measurements of turbulent shear stresses obtained by hot-wire techniques and detailed distributions of the angular displacement between mean flow, mean shear and turbulent stress components, Ref. 28, 29.

A further important contribution is the summary of the Berlin IUTAM Symposium of 1982, Ref. 30.

10. University of Karlsruhe, Institut fur Stromungslehre und Stromungsmaschinen (K.O. Felsch)

The motivation of this group comes from the problems of flow in rotating machinery. Experimental as well as theoretical results have been obtained for internal as well as external flows. Four projects are briefly described here, of which the cylinder on a wall previuosly studied in great detail by K.C. Brown (Ph.D. Thesis, Melbourne University 1971) is a useful data base.

10.1 Experimental Investigation of the Pressure-Velocity Correlation in a Three Dimensional Turbulent Boundary-Layer (Rectangular Duct)

The experimental investigation is made with a pressure-velocity probe, consisting of two pairs of closely adjacent miniature hot-wire and 1/8"-microphone, Fig 10.1. The distance between the pairs is variable, making spatial as well as time correlation measurements of pressure and velocity fluctuations possible. The data acquisitiion is done by a digital computer which enables one to calcualte all correlations from one ensemble. By means of digital filter procedures, the separation of micro-and macro-scale characteristics is possible, Ref. 31.

10.2 Flow in Rotating Channels

The fully developed turbulent flow in channels with parallel walls and rectangular cross section is calculated. The cases of a curved channel, a channel which rotates about an axis perpendicular to the main flow direction, and finally a curved rotating channel are treated. The flow is calculated in a cross-section by solving the Reynolds equations in connection with the k-ϵ-turbulence model.
For a large range of parameter values the results agree with measurements.

10.3 Measurements of Turbulent Flow by Triple Hot-Wire Techniques (e.g. Cylinder on Flat Plate, Ref. 32, 33)

The complete measurement of a turbulent velocity field requires simultaneous measurements of the three instantaneous velocity components. The method has the following features. The velocity vector will be determined with correct sign if the direction of the u-component coincidental with the probe-axis is known to be positive. That means the evaluation technique is not intended for measurements in reversed flows. No special requirements are given to the accordance with a particular sensor orientation. It is only necessary to determine the angles of the wires relative to the probe axis before performing the data processing. The cooling characteristic is decribed by three nonlinear equations implicitly containing the Cartesian velocity components. Since this system requires an iterative search algorithm, the application of a digital computer is unaviodable. On the other hand, this is preferable for good accuracy, anyhow, because no analogue apparatus besides the anemometer itself is used. Each of the three hot-wires is oriented in a different coordinate plane and makes an angle of 45 degress to the axes. The physical consequence is that each sensor reacts most sensitively to one Reynolds shear stress and is very insensitive to the other ones. The wires are placed within a virtual sphere with a diameter of 1.8 mm (see Ref. 34).

10.4 Measurements of the Mean Velocity and of the Reynolds Stress Tensor in a Three-Dimensional Turbulent Boundary Layer induced by a Cylinder Standing on a Flat Wall.

Extensive experiments were carried out in a three-dimensional incompressible turbulent boundary-layer growing in front of a cylinder standing on a flat wall in a wind tunnel. With the method of streamline-orientated X-wires, profiles were measured of the mean velocity vector, the 6 components of the turbulent stress tensor and the static pressure and the wall shear stress vector. Dechow and Felsch's, Ref. 35, measurements along a streamline are now supplemented by measurements perpendicular to the streamlines, Ref. 36.

11. Technical University of Aachen, Aerodynamisches Institut (E. Krause)

Experimental as well as theoretical work of a fundamenal nature is proceeding. A brief description of the group's research is presented below.

11.1 Experimental Investigations

a) Measurements: An incompressible turbulent boundary layer developing from two-dimensional initial conditions to a fully three-dimensional flow was investigated in detail, Ref. 37-39. Fig. 11.1. describes the geometry of the experimental set-up. The flow is deflected by two flat deflector plates towards and laterally across the flat plate on which the boundary layer has grown. Also shown in Fig. 11.1 are measured and computed mean velocity profiles at selected stations. The corresonding Reynolds stresses are given in Fig. 11.2. Fig. 11.3 is a photograph of the wall streamlines as visualized by surface oil flow. The profiles of the mean velocities and the Reynolds stresses were measured at 21 stations with hot-wire anemometry. The wall shear stresses were inferred from Preston tube measurements as well as from Clauser charts. The pressure distribution was mapped out on a fine grid at the outer edge of the boundary layer. Based on the measurements, near-wall similarity of the mean flow as well as several assumptions used for turbulence modelling were checked for their validity in this flow. The results formed the basis for a suggestion for joint European Experiments in which turbulent flow properties will be determined in three-dimensional boundary layers with improved measuring techniques. The details are described in Ref. 40.

b) Development of hot-wire measuring techniques: Special attention was devoted to determing the accuracy of hot-wire measurements. The limits of standard hot-wire techniques were discussed in Ref. 41 and an improved data reduction method is also described therein. Presently, a new hot-wire technique for measuring instantaneous velocity vectors in low and high-intensity turbulent flows is under development. The results obtained so far are described in Ref. 42.

11.2 Computation of Boundary Layer Flows:

The implicit finite-difference method of Ref. 43 was used for comparision calculations with the experiments ("Trondheim Trials") in Ref. 44. The solution was also applied to a coupled viscous-inviscid flow computation for transonic wings, Ref. 45. Numerical simulations of several experiments of incompressible three-dimensional turbulent boundary layers were carried out in Ref. 46 with special emphasis on turbulence modelling (isotropic and anisotropic algebraic eddy-viscosity-type closure and 1-eg.-model). Presently other turbulence models are being incorporated in the solution.

12. University-of Hamburg, Institut fur Schiffbau (J. Kux)

Apart from two theoretical contributions, Ref. 47, 48, this group has produced comprehensive experimental results together with a very detailed analysis on the separated flow near the stern of a ship. Most of this was carried out on a double model in a wind tunnel, but isolated measurements on a real ship were made with laser-doppler anemometry in a cooperative project with the Shell Company, Ref. 49. The first wind tunnel measurements were selected as a test case at Goteborg, Ref. 50. More detailed measurements on this model followed (10,000 measuring points with a five-hole probe supplemented by LDA measurements which yielded some components of the Reynolds stress tensor), Ref. 51-53. Though these measurements do not reach the lower boundary layer, they give detailed information about the outer part.

The evaluation of this set of data has shown that it is possible to determine the vorticity field. This is clearly an important data base, especially for separated flows. To give an impression of the quality of the results, Figure 12.2 shows the distribution of the three components of velocity and of the three components of vorticity a long a line normal to point 7 on the ship hull shown in Fig. 12.1. The magnitude of the vorticity, which has reached approximately 40/m at the point nearest the wall continues to rise to approximately ten times this value at the wall. (Determined by Preston tube, Clauser plot and Ludwieg-Tillmann formula).

13. University of Karlsruhe, Institut fur Hydromechanik (W. Rodi)

The Patankar-Spalding, Ref. 55, finite difference procedure for solving 3D parabolic flow equations was used together with the K-ϵ turbulence model to calculate a number of different 3D boundary layer flows. The application to a boundary layer on a flat plate approaching a circular cylinder mounted on the plate, the boundary layer beneath the leading-edge vortex on a delta wing and the boundary layer on the upper surface of a curved duct with both zero and adverse longitudinal pressure gradients is reported in Ref. 56. The same method was also applied to the 3D boundary layer on a swept wing as reported in Ref. 57. In all these cases a simplified version of the Patankar-Spalding scheme was used in which the pressure was prescribed by the free-stream velocity and the equations were parabolic in the two directions parallel to the wall. In general, satisfactory results were obtained, but in its present version the method can only yield results for the region upstream of the cross section at which separation first occurs. The use of an isotropic eddy viscosity, which, according to a number of experiments, is unrealistic and leads to incorrect shear stress directions, does not seem to impair seriously the predictions of quantitities of engineering interest, at least for the situations investigated. A similar conclusion can be drawn from the calculations of the flow along a wing-body junction which were carried out for the Stanford Conference on Complex Turbulent Flow, Ref. 58. In this case, the lateral skewing of the streamwise vorticity in the corner flow, and the calculation of the decay of this vortex was of primary interest. Here the pressure field had to be calculated and the equations were parabolic only in the main-flow direction. The agreement between calculated and measured distribution of friction coefficient was found to be surprisingly good.

Calculations were also carried out for corner flows in which the secondary motion is driven by the turbulence. The development of the boundary layers in the corner of a rectangular duct was calculated starting from a uniform velocity profile at the entrance, Ref. 59. In these calculations, the interaction of the boundary layers developing on the two perpendicular walls was simulated as well as the setting-up of the turbulence-driven secondary motion. This motion cannot be simulated with an isotropic eddy viscosity model as used in the calculations described above. Various versions of an algebraic stress model were employed which were developed by simplifying a higher-order model solving transport equations for the individual Reynolds stresses. This contribution is supplemented by Fig. 13.1.

14. DFVLR Institute for theoretical fluid dynamics, Gottingen (W. Krodulla)

Calculation methods based on finite difference techniques have been developed and applied to the solution of flows over an ellipsoid of revolution, Ref. 60, and laminar flow over wings, Ref. 61. An example of the latter is presented in figures 14.1 and 14.2. Work is preceeding on the computation of separated transonic flow over a hemisphere-cylinder and slender supersonic bodies by Reynolds-averaged Navier-Stokes solution. In conjunction with the experimental institute (see 15) considerable effort is spent on research into the mechanisms of instability of a three-dimensional boundary layer.

15. DFVLR Institute for experimental fluid dynamics, Gottingen (H.U. Meier)

For a number of years, this group had been building up a growing data base of experimental results on the flow over a 6:1 ellipsoid of revolution. The measurements include steady and fluctuating wall shear stress in magnitude and direction (hot film) and surface flow fields visualized by oil flow and supplemented by liquid crystal visualization. Boundary layer investigations include pitot probe surveys and initial results from hot wire and laser-doppler measurements. The outer flow field has been investigated in some detail with a ten-hole probe. The investigation covers a wide range of Reynolds numbers and angles-of-attack. Visualization of the spatial flow field, an important component of such an investigation, because it shows the critical regions at a glance, was performed by light-sheet smoke visualization as well as the hydrogen bubble tecnhique and dye flow. The results of this work have lead to a cooperation with the ONERA, under which new experiments have been made in the F1 Wind Tunnel at Toulouse as well as in S2 at Chalais-Meudon. Further experiments are planned in the DNW in the summer of 1984. As it becomes more complete, this data base is becoming more and more attractive to potential computors. Figures 15.1 and 15.2 show representative results.

16. Concluding Remarks

In the preceding pages an attempt has been made to present the work of the various groups in Germany who are doing research on three-dimensional boundary layers. More than anything else this attempt has led to a realization of the need to coordinate the work and of the many gaps which remain in our knowledge of the phenomena. If this round table discussion leads to a way of improving the coordination, much will have been achieved.

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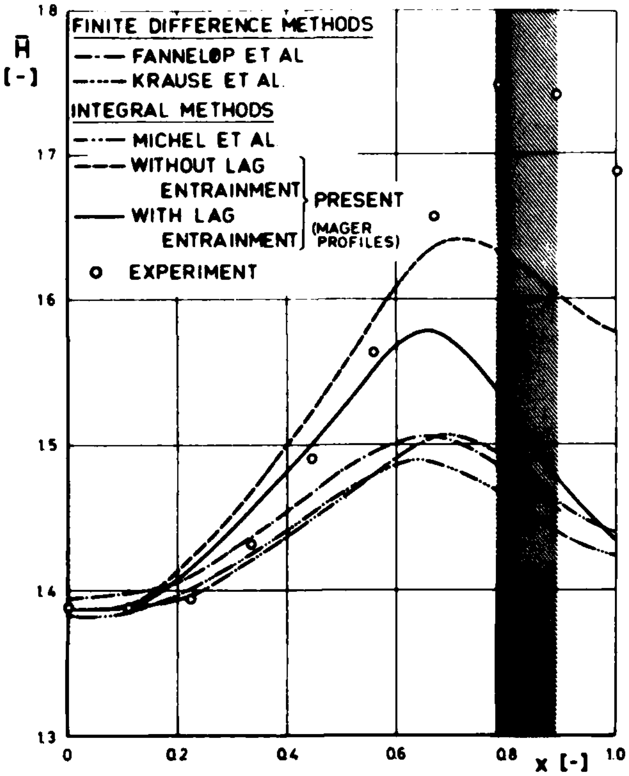


Figure 5.1 Shape Parameter Distribution (Berg And Elsenaar Test Case)

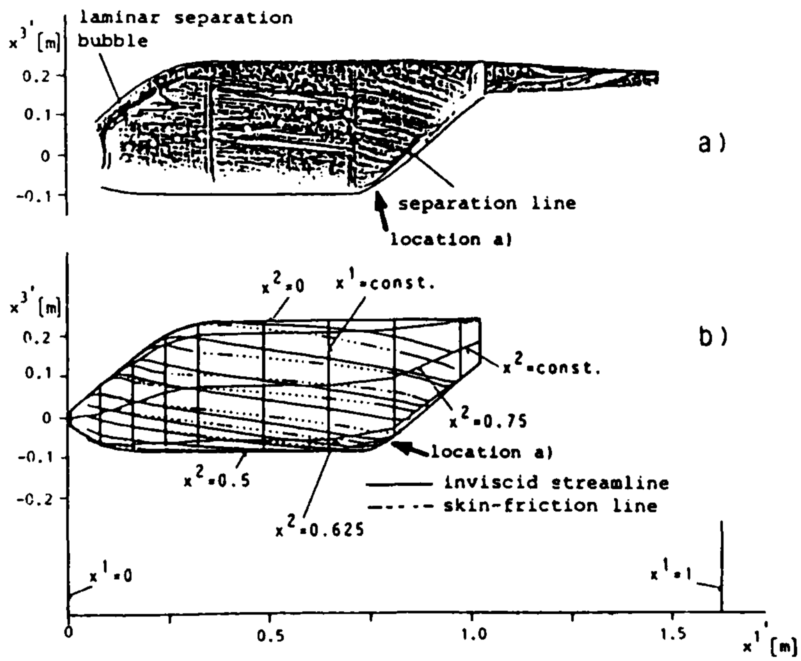


Figure 6.1 Side view of helicopter fuselage (left half) [4]:
a) oil flow picture;
b) computed inviscid streamlines and skin-friction lines
(Re = $6.6\cdot 10^6$, M_∞ = 0.184, α = $-5°$)

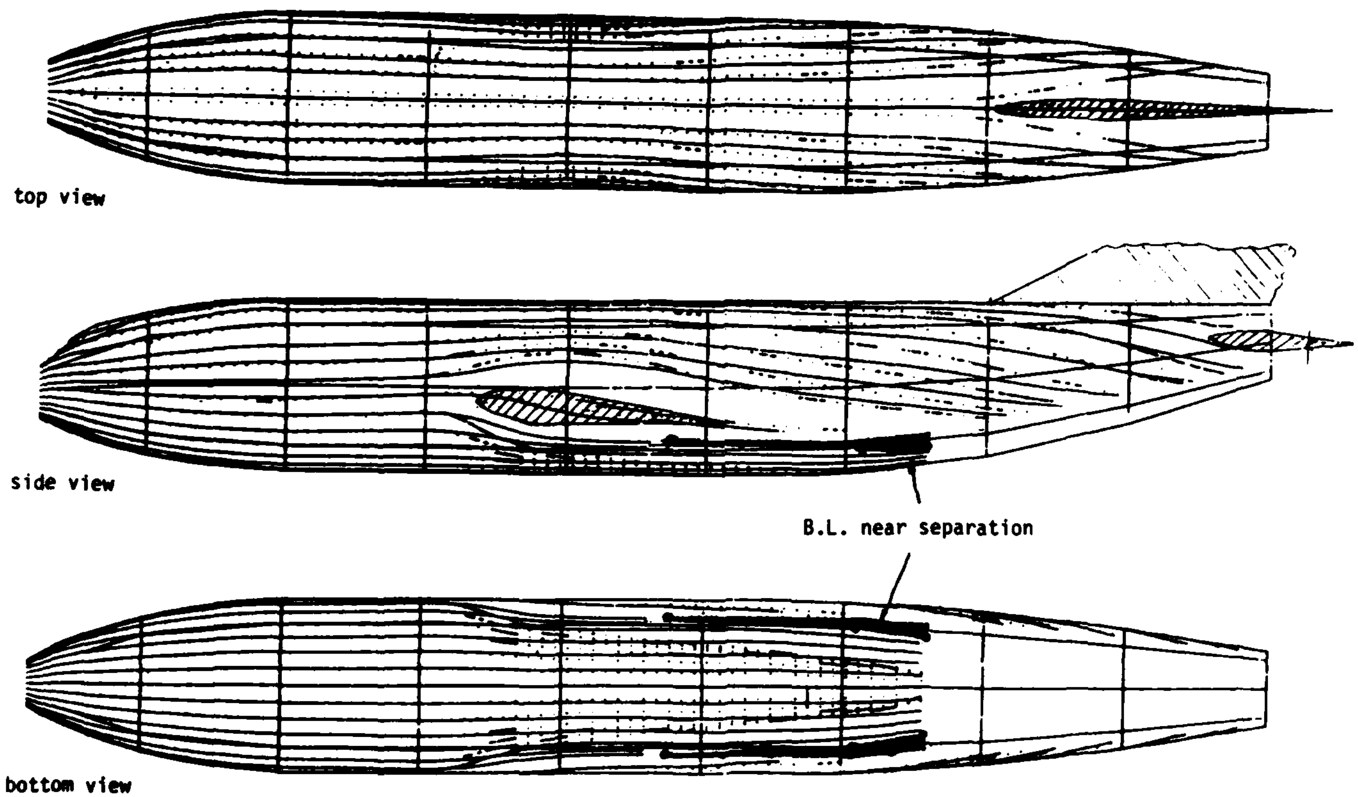
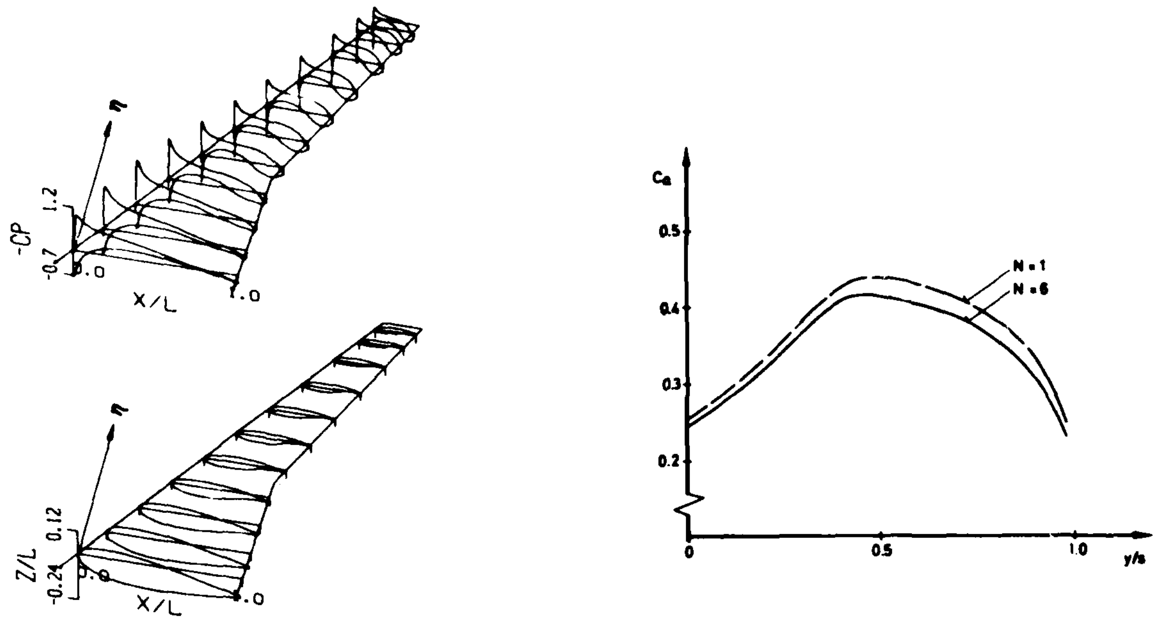


Figure 7.1 Potential And Wall Streamlines On An A310 Fuswlage; M = 0.8, Alpha = 0.6

B10-2V-Flugel $\Lambda = 8.0$, $\varphi_{0.25} = 28°$, $\lambda = 0.329$
$Re_{l_\mu} = 3.4_{10}7$, $\alpha = 0°$, $M_\infty = 0.2$



Flugelstromung: Panel/ GS-Lösung gekoppelt (N=6)
Druckverteilung und Geometrie

Flügelströmung: Panel/ GS-Lösung gekoppelt (N=6)
Auftriebsverteilung längs Spannweite,
bezogen auf lokale Flügeltiefe

Figure 7.2 Inviscid/B.L. Coupling On A 3-D Wing

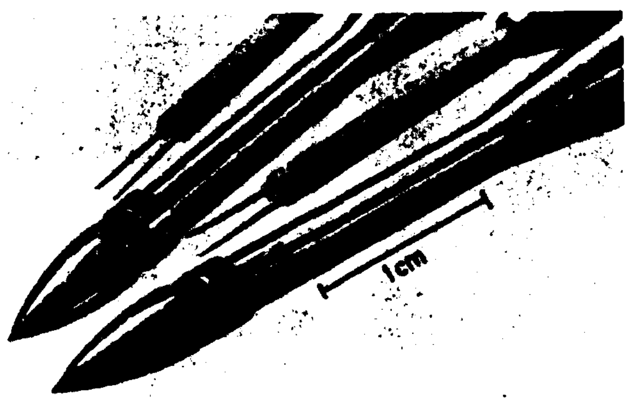


Fig. 10.1 Pressure velocity correlation probe.

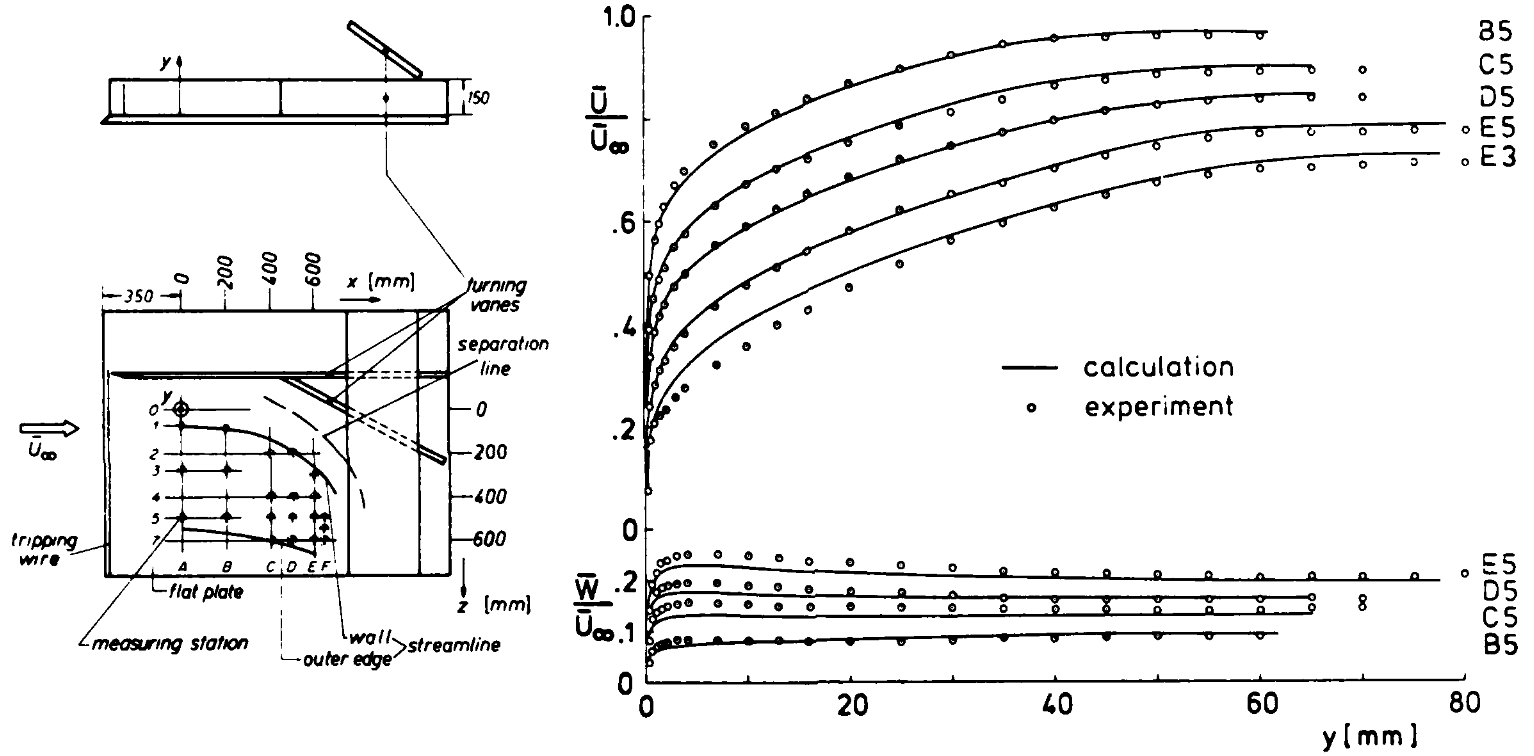


Figure 11.1 Experimental Setup For 3DTBL; Measured And Computed Mean Velocities

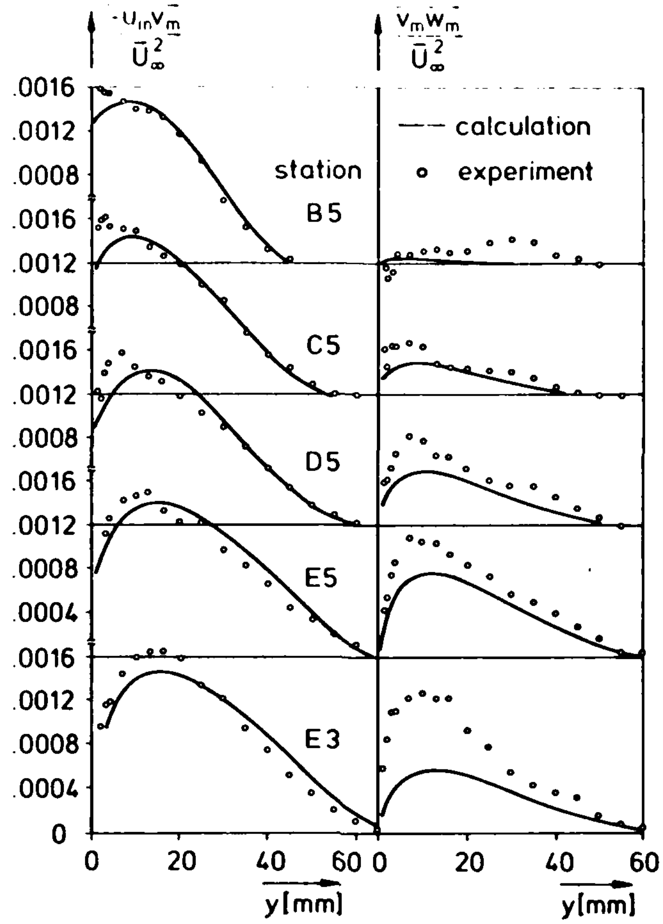


Figure 11.2 Measured And Computed Reynolds Shear Stresses

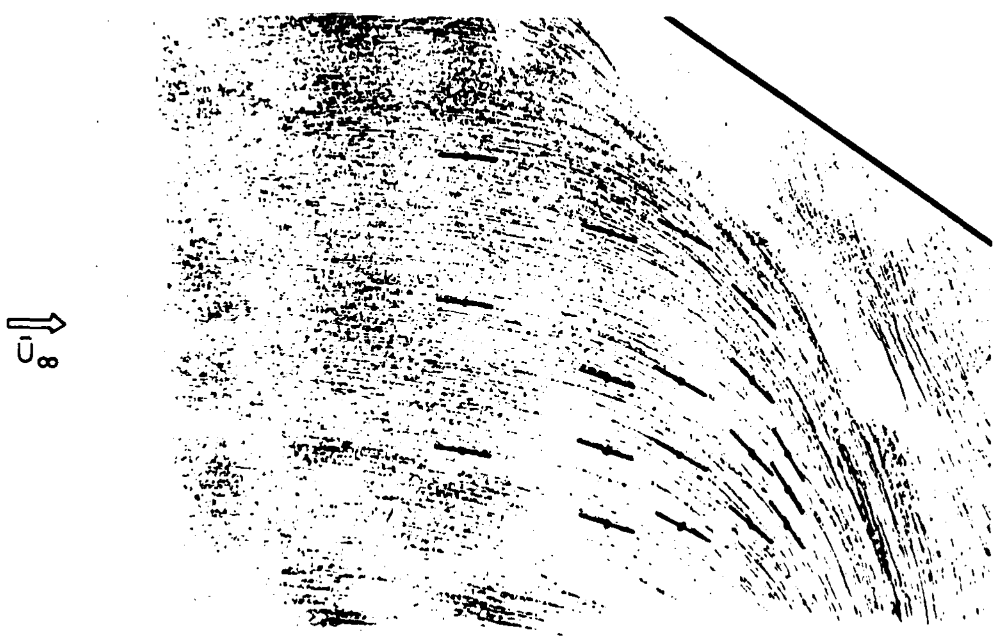


Figure 11.3 Oil-Flow Pattern And Computed Wall-Streamline Directions

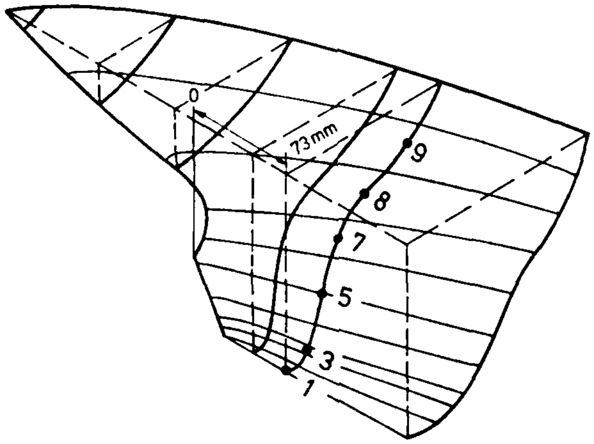
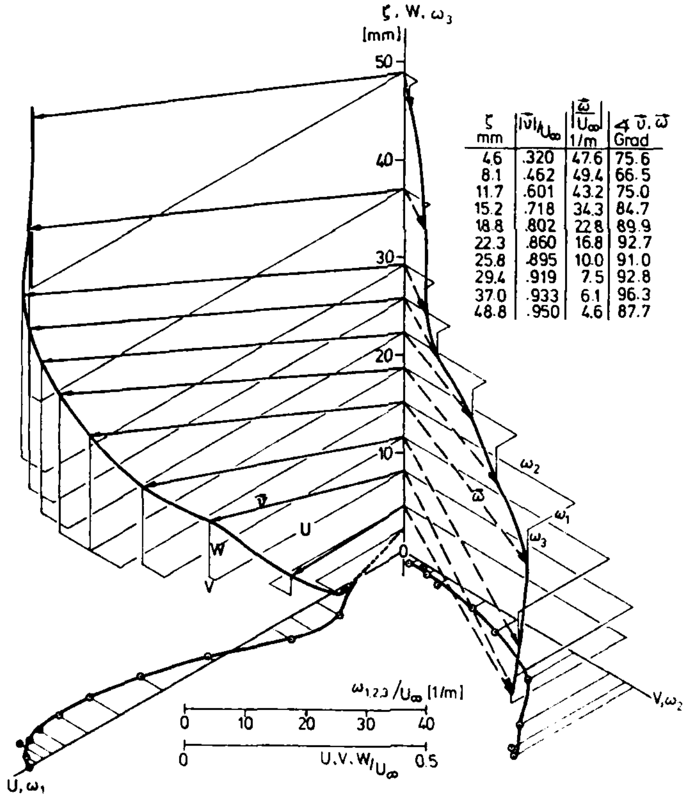


Figure 12.1 Experimental Ship-Hull Model Stern Geometry



ζ mm	$\lvert\vec{v}\rvert/U_\infty$	$\lvert\vec{\omega}/U_\infty\rvert$ 1/m	$\sphericalangle\ \vec{v},\vec{\omega}$ Grad
4.6	.320	47.6	75.6
8.1	.462	49.4	66.5
11.7	.601	43.2	75.0
15.2	.718	34.3	84.7
18.8	.802	22.8	89.9
22.3	.860	16.8	92.7
25.8	.895	10.0	91.0
29.4	.919	7.5	92.8
37.0	.933	6.1	96.3
48.8	.950	4.6	87.7

Figure 12.2 Velocity And Vorticity Distributions At Points On A Line Normal To Point 7 In Fig. 12.1

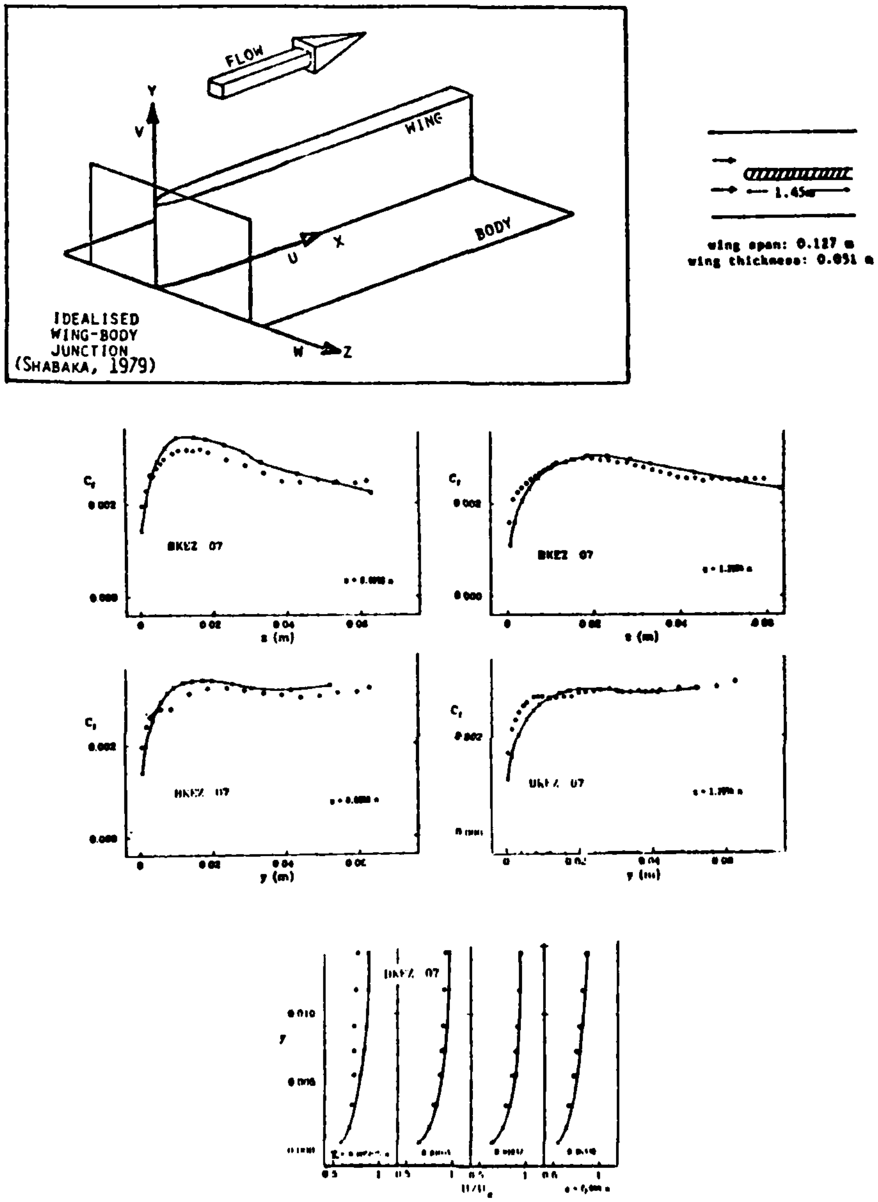
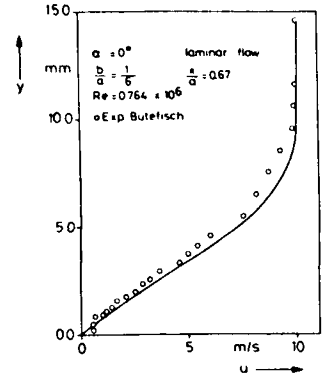
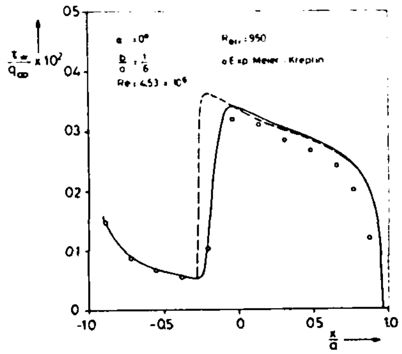


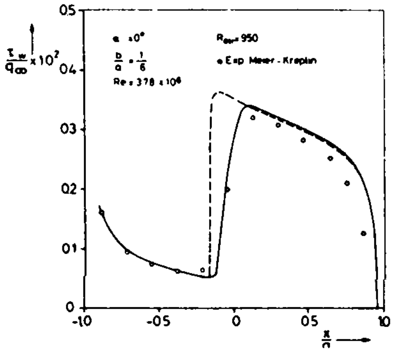
Figure 13.1 Calculation Of Flow In The Corner Of A Wing-Body Junction
(From: Proc. Of 1980-81 Stanford Conference On Complex Turbulent Flows, Vol. III, pp. 1235-1236)



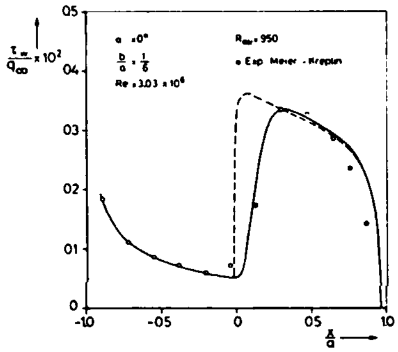
Laminar velocity profile at x/a = 0.67



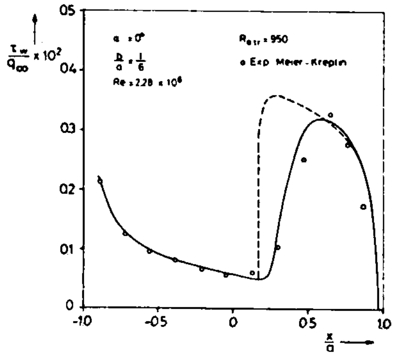
Wall shear stress on a prolate spheroid 1:6, α = 0°



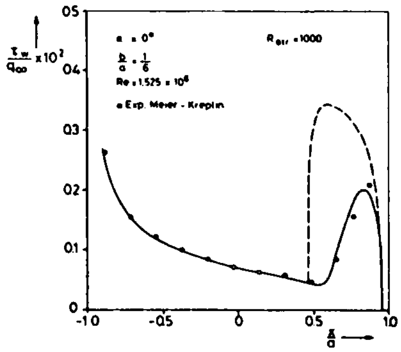
Wall shear stress on a prolate spheroid 1:6, α = 0o



Wall shear stress on a prolate spheroid 1:6, α = 0°

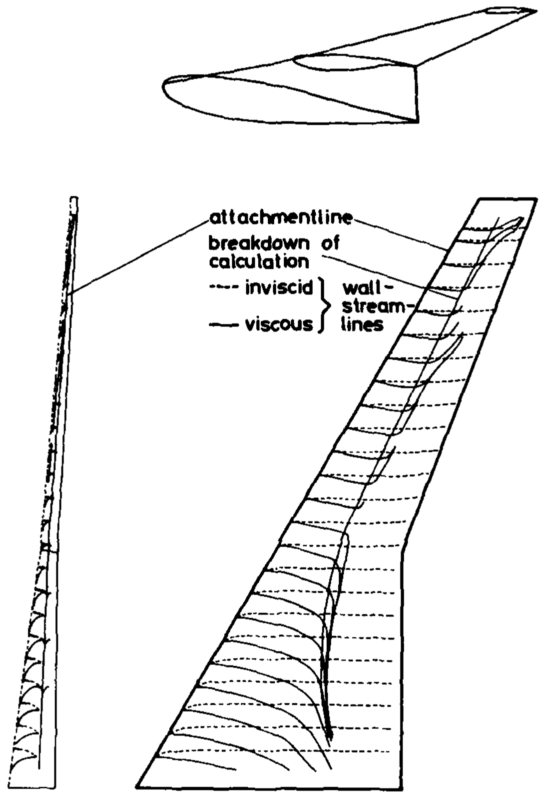


Wall shear stress on a prolate spheroid 1:6, α = 0°



Wall shear stress on a prolate spheroid 1:6, α = 0°

Figure 14.1 Computed Results For Flow Over A Prolate Spheriod (Ref. 60)



Streamlines on the lower surface of the wing

Figure 14.2 Computed Results For Flow Over A Wing (Ref. 61)

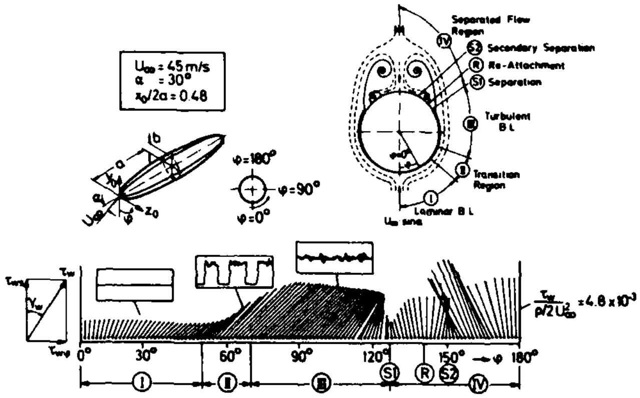


Figure 15.1(a) Measured wall shear stress vectors in the central cross-section, $x_0/2a = 0.48$, at $\alpha = 30°$, $U_\infty = 45$ m/s and a systematical sketch of the flow field around a prolate spheroid at high incidence

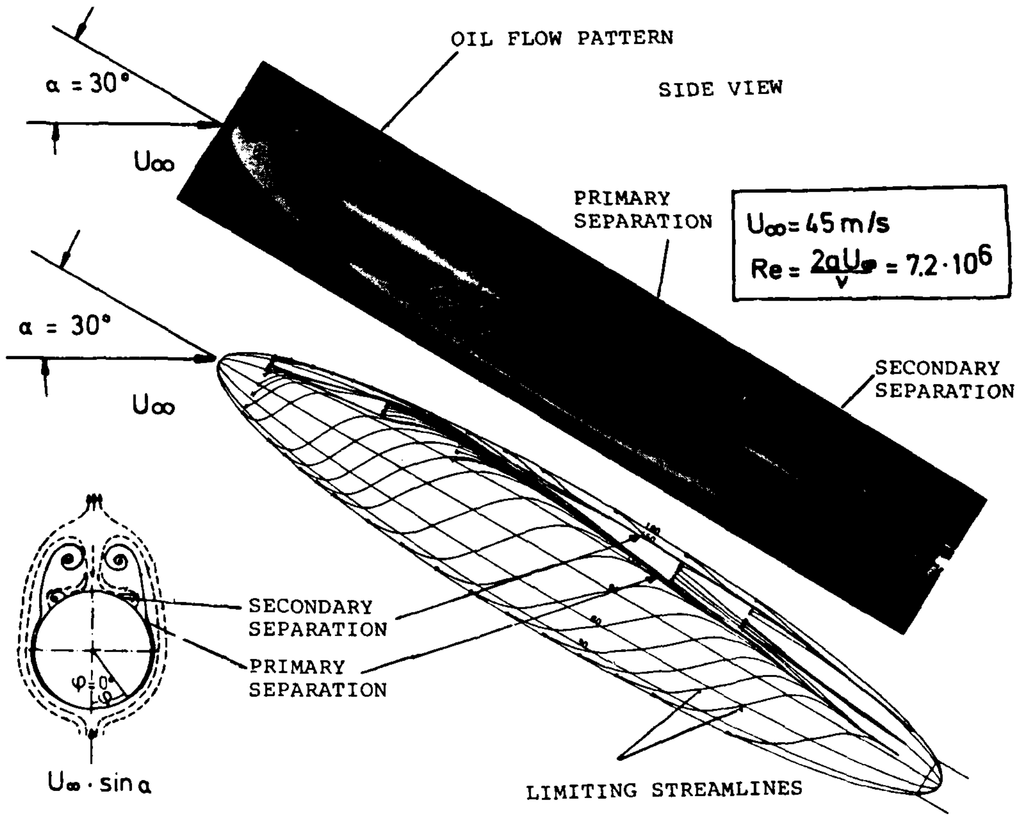
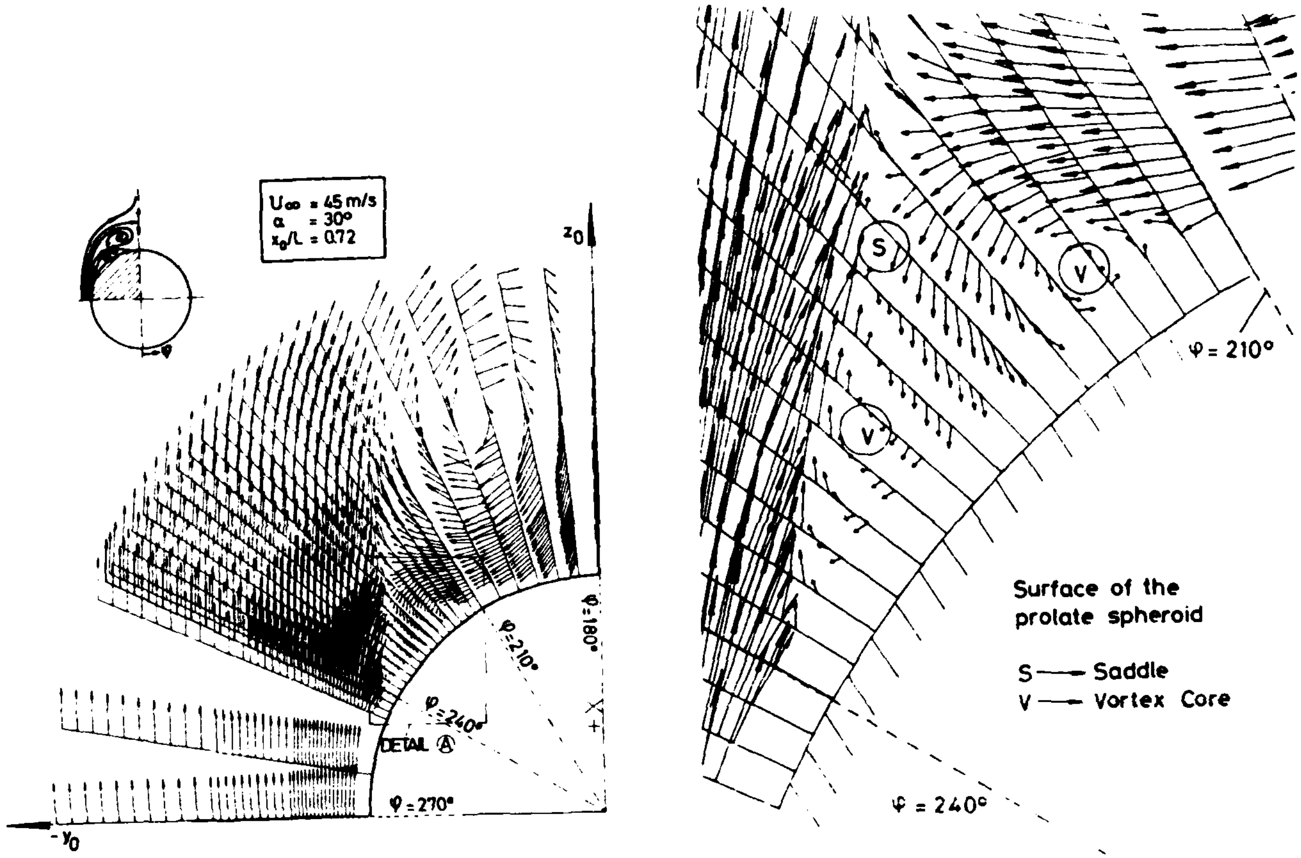


Figure 15.1(b) Comparison of an oil flow pattern with limiting streamlines obtained from wall shear stress measurements



Detail A

Figure 15.2 Cross-Flow Velocity Distribution In The Cross Section X_0/L = 0.72

THREE-DIMENSIONAL BOUNDARY LAYER RESEARCH AT NLR

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SUMMARY

A review is given of the experimental and theoretical research work on three-dimensional boundary layers, carried out at NLR between 1970 and 1984, or presently under development.

LIST OF SYMBOLS

c	wing chord
c_f	skin friction coefficient
U	velocity
x,y,z	coordinates
α	angle of attack
δ	boundary layer thickness
δ_1^*	streamwise displacement thickness
θ_{11}	streamwise momentum thickness
ν_e	eddy viscosity
ρ	density
τ	shear stress
φ	flow angle

Subscripts

c	component normal to local flow direction
e	value at the boundary layer edge
s	component parallel to local flow direction
w	value at the wall
z	component normal to external flow direction

1. INTRODUCTION

Theoretical work on three-dimensional boundary layers started already at NLR in the fifties (e.g. Ref. 1). At that time, however, the computational efforts involved in three-dimensional boundary layer calculations were beyond the practical possibilities. The advent of the modern computer altered the situation essentially, giving a new momentum to the activities in the field. At NLR a concentrated effort on three-dimensional boundary layer research was taken up again in 1970. The research comprised both experimental and theoretical work. The present paper gives a review of the work done from that time on. The re-

view includes the research done in the early seventies because the results of that work still guide the more recent developments. In the recent work emphasis has been laid on the development of a practical calculation method for three-dimensional laminar and turbulent boundary layers. Results of various applications of the calculation method will be discussed. A progress report of work under development will be given and finally some problems that remain to be solved will be mentioned.

2. EARLY EXPERIMENTAL AND THEORETICAL WORK

In 1970 an experimental research program on three-dimensional turbulent boundary layers was initiated and simultaneously a start was made with the development of a three-dimensional turbulent boundary layer calculation method. The aim of the experimental work was specifically to provide a check for the calculation method. To make comparisons with calculations easy, a quasi-two-dimensional flow, as occurs on an infinite swept wing, was simulated in the experiment. Such a boundary layer flow was generated on a 35° swept flat plate, using a pressure-inducing body, so-called guiding vanes and a blockage at the end of the channel formed by the vanes, the body and the plate, as sketched in figure 1. The model design is described in more detail in reference 2.

At measuring station 1 the turbulent boundary layer is very nearly two-dimensional. Downstream of station 1 the boundary layer becomes three-dimensional due to the adverse pressure gradient on the swept plate. Figure 2 shows some experimental results. The difference between the wall flow angle (or skin friction direction) φ_w, and the flow angle at the boundary layer edge, φ_e, is seen to increase fast in downstream direction. Near measuring station 9 the wall flow angle exceeds $\varphi_w = 55°$, which means that the wall streamline is parallel to the plate leading edge or the line of invariancy in this quasi-two-dimensional flow. At that position the three-dimensional boundary layer is said to be separated. As appears from figure 2 the skin friction magnitude is not zero at the separation line, but is close to a minimum there. More experimental data, including the results of boundary layer velocity measurements with a rotatable hot-wire anemometer, are contained in reference 2.

In the mean time a calculation method for three-dimensional incompressible turbulent boundary layers on developable surfaces had become available at NLR (Ref. 3). The method is an extension of the two-dimensional calculation method due to Bradshaw (Ref. 4), one of the better methods at that time. This is a so-called field method, which solves the equations of motion themselves. In that case the only assumptions to be made are about the turbulent shear stresses. Introducing a turbulent shear stress vector parallel to the wall, $\vec{\tau}$, the turbulent shear stress transport equation reads:

$$\frac{D\vec{\tau}}{Dt} = |\vec{\tau}| \frac{\partial \vec{U}}{\partial y} - \frac{\partial}{\partial y} \{V_d \vec{\tau}\} - \left\{\frac{|\vec{\tau}|}{\rho}\right\}^{\frac{1}{2}} \frac{\vec{\tau}}{L}$$

where the diffusion velocity V_d and the dissipation length L are empirical algebraic functions. The equations are solved with a finite-difference method. The turbulent shear stress transport equation employed is not valid in the viscous sublayer close to the wall. In the wall region, therefore, the flow is not computed with a finite-difference method, but is described by an analytical function: the law of the wall.

A comparison between the experimental data and the calculation results with the method described above is given in figure 3. Two laws of the wall have been employed in the calculations: the normal two-dimensional law of the wall and a three-dimensional law of the wall, which takes into account the rotation of the velocity vector in the wall region (Ref. 5). It is evident from the results that it is essential to take into account the three-dimensionality of the flow in the wall region. Only in that case the calculation results are within certain limits independent of the choice of the wall distance, y_1, where the law of the wall and the finite-difference calculations are matched. Even when using a three-dimensional law of the wall, however, good agreement between calculations and measurements is only obtained up to station 5. Further downstream increasing deviations are seen to occur. Notably the calculations do not predict the three-dimensional separation observed in experiment.

The experiment was used as a test case for calculation methods at Euromech Colloquium 60, Trondheim in 1975 (Ref. 6). It appeared that similar deviations from experiment were evident for most calculation

methods, such as those using the popular algebraic eddy viscosity turbulence model. Actually the differences between the predictions using Bradshaw's turbulence model and the more simple algebraic eddy viscosity model are small in this case.

To investigate the reason for the discrepancy between calculation and experiment further, turbulence measurements with hot wires were carried out in the three-dimensional boundary layer. The results of the turbulence measurements are reported in reference 7. For the interpretation of the results it is convenient to deduce an eddy viscosity from the measured turbulent shear stress and the local velocity gradient:

$$\nu_e = \frac{|\vec{\tau}|}{|\partial\vec{U}/\partial y|}$$

The thus deduced eddy viscosities at a number of measuring stations are plotted in figure 4. The normal value assumed in algebraic eddy viscosity turbulence models is also indicated. It is evident that at the downstream stations the measured eddy viscosities are essentially smaller than that value. The measurements also showed that the direction of the turbulent shear stress does not coincide with the direction of the velocity gradient. This means that the eddy viscosity is non-isotropic according to the measurements. A non-isotropy ratio can be defined as follows:

$$N = \frac{\tau_c/(\partial U_c/\partial y)}{\tau_s/(\partial U_s/\partial y)}$$

where the subscripts s and c refer to the direction parallel and normal to the local flow direction respectively. The non-isotropy ratios deduced from the measurements are plotted in figure 5. The crosswise eddy viscosities are seen to be essentially smaller than the streamwise eddy viscosities, i.e. the crosswise shear stresses are comparatively small. It can be concluded that the turbulence measurement results suggest that there is a reason for the poor predictions of the calculation methods, as they seem to overestimate the turbulent shear stresses, especially the crosswise component.

The above conclusion also holds for Bradshaw's turbulence model, because the difference from an algebraic eddy viscosity model assumption is small here. To investigate the role of turbulent shear stresses, further calculations have been made with a modified version of Bradshaw's turbulence model. In fact the model was so adapted that the measured turbulent shear stresses were reproduced in the calculations. The results are depicted in figure 6. With the adapted turbulence model good agreement with experiment is seen to exist up to station 7. This means that up to that station the deviations between calculations and measurements found earlier are to be attributed indeed to wrong turbulent shear stress predictions in the calculations.

Also with the adapted turbulence model significant deviations from experiment are still apparent downstream of station 7 and again no separation is predicted. Figure 6 also includes calculation results obtained with the surface pressure increased at stations 8 and 9 with 0.5 and 1.0 % of the dynamic head respectively. With the slightly larger pressure gradient separation is now predicted early. The result suggests singular behaviour of the solution of the equations at separation, as for two-dimensional boundary layers (Ref. 9). To avoid singular behaviour the interaction between the inviscid outer flow and the boundary layer flow should be taken into account in the calculations. To investigate this point further some preliminary calculations with an interaction model were subsequently made (Ref. 8). A simple interaction model is possible here because the inviscid flow may be regarded as a quasi-one-dimensional channel flow (Fig. 1). The results of these calculations are shown in figure 7. Good agreement between experiment and calculations is now seen to exist up to close to the three-dimensional separation line, when using the adapted turbulence model. Note that with the original turbulence model the agreement starts to degrade downstream of station 5, but that separation is predicted, though much too far downstream as compared with experiment.

The results of the theoretical and experimental work described in this section, comprising the research done in the period 1970-1975, can be summarized as follows. In the first place it appeared that the three-dimensionality of the flow in the wall region of the boundary layer can not be neglected and

should be taken into account in calculation methods. Further, the experimental results clearly showed that the current turbulence models predict the turbulent shear stresses in three-dimensional boundary layers with poor accuracy. Finally it was found that calculation results near a three-dimensional separation line are dominated by the viscous-inviscid interaction as in two-dimensional boundary layers near separation.

3. DEVELOPMENT OF A PRACTICAL CALCULATION METHOD

The three-dimensional boundary layer calculation method available at NLR since 1971 (Ref. 3) was applied some years later to a new supercritical wing design. The turbulent boundary layer flow on the root section of the wing upper surface was computed. The calculated wall streamlines are depicted in figure 8. The calculations showed the presence of a line of convergence, suggesting a three-dimensional so-called open separation at that position. Subsequently the wing model was tested in the wind tunnel. The oil flow picture obtained in the tests, also shown in figure 8, confirms the presence of an open separation, which in practice appears to lead to a strong vortex flow there. The more two-dimensional separation region predicted further to the right is visible too in the oil flow pattern. It may be concluded that useful results were obtained with the calculation method.

Though the results were satisfactory, it became also clear that the application of the calculation method is very time consuming, both from the user and from the computer. Since three-dimensional boundary layer calculations on a routine basis should become feasible, the decision was taken to develop a new, more users friendly and faster calculation method. The new method should also be more general applicable by extending it to compressible laminar and turbulent boundary layers on developable as well as non-developable surfaces. The main features of the new method (Refs. 10, 11) are:

- An arbitrary non-orthogonal curvi-linear surface coordinate system can be chosen by the user.
- The organization of the calculation method has been designed with the emphasis on the users-oriented properties.
- An efficient finite-difference method has been employed for the whole boundary layer, including the wall layer.
- An algebraic eddy viscosity turbulence model has been used, because the model is simple, while for boundary layer flows no other current turbulence models do a significantly better job. The eddy viscosity equation reads:

$$\nu_e = F^2 \ell^2 \left| \frac{\partial U}{\partial y} \right|$$

  where F is a wall damping function and ℓ is the mixing length, being a function of the wall distance and the boundary layer thickness.

Initial conditions of the boundary layer are supposed to be given along a line on the surface. Downstream of the line segment with initial data one may distinguish a region of influence and a region of determinacy, see figure 9. The latter region coincides with the domain which can be computed in downstream marching boundary layer calculations with the given initial data. The new calculation method computes the complete computable domain. If a separation region is present, the calculation circumvents that region, as it is no part of the region of determinacy, but the computations are proceeded around it.

Depending the circumstances, various difference molecules are employed in the finite-difference computations. Special difference molecules are, of course, necessary at the free side boundaries of the computational domain, but actually a larger variety of difference molecules is applied dependent on the direction of the cross flow, see figure 10. An algorithmic table is constructed and based on the information in the table, difference molecules and a scanning sequence are selected. The scanning sequence is optimized to alleviate as much as possible step size limitations from numerical stability requirements. Due to these features a flexible and efficient calculation method has been obtained.

In the following some applications of the new three-dimensional boundary layer calculation method to practical aircraft configurations will be discussed. One example is the application of the boundary layer calculation method as a part of a system for the numerical simulation of the transonic flow around wing-body configurations (Ref. 12). The inviscid flow is computed with a full potential method. The surface pressure distribution obtained is then employed as an input for the boundary layer calculations. The calculations were started at 5 % chord behind the wing leading edge with estimated initial conditions. A typical standard graphics output of the system is shown in figure 11. The computed wall streamlines on the wing upper surface are plotted. The result may be regarded as a "numerical oil flow picture". Note that the computed wall streamlines do not extend to the wing trailing edge. This is because no viscous-inviscid interaction has been taken into account, so that separation is computed upstream of the trailing edge. Though the computational system can still be improved in many respects, it has proven already now to be a valuable tool in transonic wing design.

In the above example the boundary layer calculation method is used on routine basis as a (small) part of a computational system. A similar system exists for the calculation of the potential and boundary layer flow around engine cowls. Figure 12 gives a typical result (Ref. 13). The boundary layer calculations were started a short distance behind the engine cowl front with estimated initial conditions. The boundary layer calculation results generally appear to be not very sensitive in practice to the initial conditions, except for the initial external flow angle (Ref. 13). Fortunately this flow angle can be obtained from the potential flow solution. The results in figure 12 indicate a region of convergence, especially of the wall streamlines, at the rear on the external surface of the engine cowl.

In the boundary layer calculations just discussed, the initial conditions had to be estimated. In principle this is not necessary if the calculations are started at the origin of the boundary layer flow. On a wing-body configuration, for instance, this is at the stagnation point on the nose of the body. The boundary layer then flows over the body, the wing-body fairing, and continues its course via the wing leading edge along wing upper and lower surface to the trailing edge. As an excercise, such a boundary layer flow was computed for a simple symmetric wing-body configuration at a small angle of attack. The calculations were not started really at the stagnation point, but comprised the turbulent boundary layer flow between body section S and wing section D, as indicated in figure 13. The figure shows the computed development of the wall flow angle, φ_w, and boundary layer momentum thickness, θ_{11}, along the "leading edge", i.e. the front of the horizontal symmetry section of the configuration.

The wall flow angle with respect to the symmetry line is seen to be small on the body between S and B, to increase substantially on the fairing between B and C and to remain large beyond C on the wing leading edge. The boundary layer thickness decreases enormously on the fairing between B and and C, as one would expect. Though the three-dimensional boundary layer flow is very complicated here, the calculations appeared well feasible. The main difficulties were due to the large changes in flow direction and, therefore, in preferred marching direction for the boundary layer calculations. For applications on routine basis the choice of a suitable boundary layer calculation grid should be automated.

As a final example the results of boundary layer calculations on the leeward side of a slender delta wing at angle of attack will be discussed (Ref. 14). The inviscid flow was computed first, taking into account the leading edge vortex flow separation (Ref. 15). A secondary flow separation occurs on the wing due to viscous effects. The boundary layer calculations, using the computed inviscid flow surface pressure distribution, predict indeed such a secondary flow separation, as shown in figure 14. As indicated the boundary layer calculations had to be split into two parts with different marching directions. Figure 14 includes an oil flow picture of observed wall streamlines. Good qualitative agreement with the calculation results is seen to exist. The boundary layer was in this case laminar, while all the other examples discussed in this section concerned turbulent flow.

On the basis of the experience obtained, it is believed that the new three-dimensional boundary layer calculation method can be regarded indeed as a practical calculation method, suitable for a great variety of flows. The applicability of the method is still limited, of course, to viscous flows for which the boundary layer approximation holds and for which the interaction between the inviscid and viscous flow is

sufficiently weak to compute both flows in a simple iterative way.

4. FURTHER DEVELOPMENTS AND REMAINING PROBLEMS

One of the conclusions in section 2 has been that boundary layer calculations near a three-dimensional separation line become very sensitive to the prescribed surface pressure distribution. This occurs because the problem is ill-posed. It is known for quite some time already that two-dimensional boundary layer calculations break down near a separation line if the surface pressure distribution is prescribed (Ref. 9). The basic reason for the breakdown of the calculations is the dominant role of the interaction between the viscous and inviscid flow near separation. Consequently normal iterative calculation procedures, using subsequent inviscid flow calculations with an allowance for the boundary layer displacement effect and boundary layer calculations at a given pressure distribution, are not effective. The viscous-inviscid interaction has to be taken into account in a more advanced way in these regions.

Strong viscous-inviscid interactions do not only occur near two- and three-dimensional separation lines, but also in other regions, for instance near airfoil trailing edges. It is important to extend the calculations to such regions. Therefore a method for the calculation of boundary layers with strong viscous-inviscid interaction is being developed at NLR. In this method the boundary layer calculations are not carried out for a prescribed pressure, but with an interactive boundary condition (Refs. 16, 17). This boundary condition takes into account in an approximate way how the inviscid flow reacts on the presence of the boundary layer. The approximate description for the interaction is based on thin-airfoil theory. For instance, the non-lifting part is written like:

$$U_{i+1}(x) = U_i(x) + \frac{1}{\pi} \int \frac{d(U_i \delta_1^*)/d\xi}{x - \xi} d\xi$$

The local inviscid flow velocity, U, is employed in this formulation instead of the surface pressure. The subscript i refers to the iteration number. The last term represents the estimated effect of the boundary layer through its displacement thickness, δ_1^*, on the inviscid flow.

The method under development at NLR is for the moment restricted to quasi-two-dimensional flows. In the near future an extension to general three-dimensional boundary layers will be undertaken. Provisionally a simple eddy viscosity turbulence model is applied. For the present calculation method the wall damping function F has been so modified that no singularity occurs at separation. To avoid discontinuous changes at airfoil trailing edges, where two boundary layers merge into a wake, a transport equation for the eddy viscosity was introduced.

Some preliminary calculation results with the method described above are depicted in figure 15. The computed boundary layer velocity profiles on a two-dimensional airfoil at angle of attack are shown. Separation occurs between station 1 and 2. No fundamental problems were encountered in the calculation of the separating boundary layer due to the strong viscous-inviscid interaction approach. It may be expected that in the near future this type of flow can be computed on a routine basis.

From the problems mentioned at the end of section 2, one has remained, namely that the turbulent shear stresses in three-dimensional boundary layers are not well predicted by the current turbulence models. The practical calculations with such an inaccurate turbulence model, discussed in section 3, do no seem to have been very much impaired by the poor turbulent shear stress predictions, however. One reason for this is that in these cases the pressure forces dominated over the shear stresses, especially in the regions where the boundary layer was three-dimensional to a significant degree. It should further be noted that no detailed comparisons with experimental data were made in these practical applications. As a demand for increased accuracy may be expected, it is unlikely that the poor state of the art in turbulence modelling will remain acceptable in the future.

In 1982 a Workshop took place at Berlin, where various three-dimensional turbulent boundary layer

calculation methods were applied to a number of experimental test cases (Ref. 18). The results of the Workshop have confirmed that the deficiences of the turbulence models still exist. Particularly the calculated crosswise shear stresses were generally far off. A typical result for one of the test cases is shown in figure 16. The crosswise shear stress plotted there is the component normal to the external streamline direction. In this test case history effects were found to play a dominant role (Ref. 19). Actually the measured turbulent shear stress appears to change scarcely over the streamwise distance considered. This behaviour is not reflected in the turbulence models applied and leads to the large deviations with experiment apparent in figure 16. Evidently here is a problem area, where much work is left to be done. In fact improvements in turbulence modelling are not only required in three-dimensional boundary layers, but as well in other complex turbulent flows (Ref. 20).

5. CONCLUSIONS

- Three-dimensional boundary layer calculations can presently be performed on a routine basis.
- In the near future three-dimensional boundary layer calculations with strong viscous-inviscid interaction will become feasible.
- Improvements in turbulence modelling are still needed.

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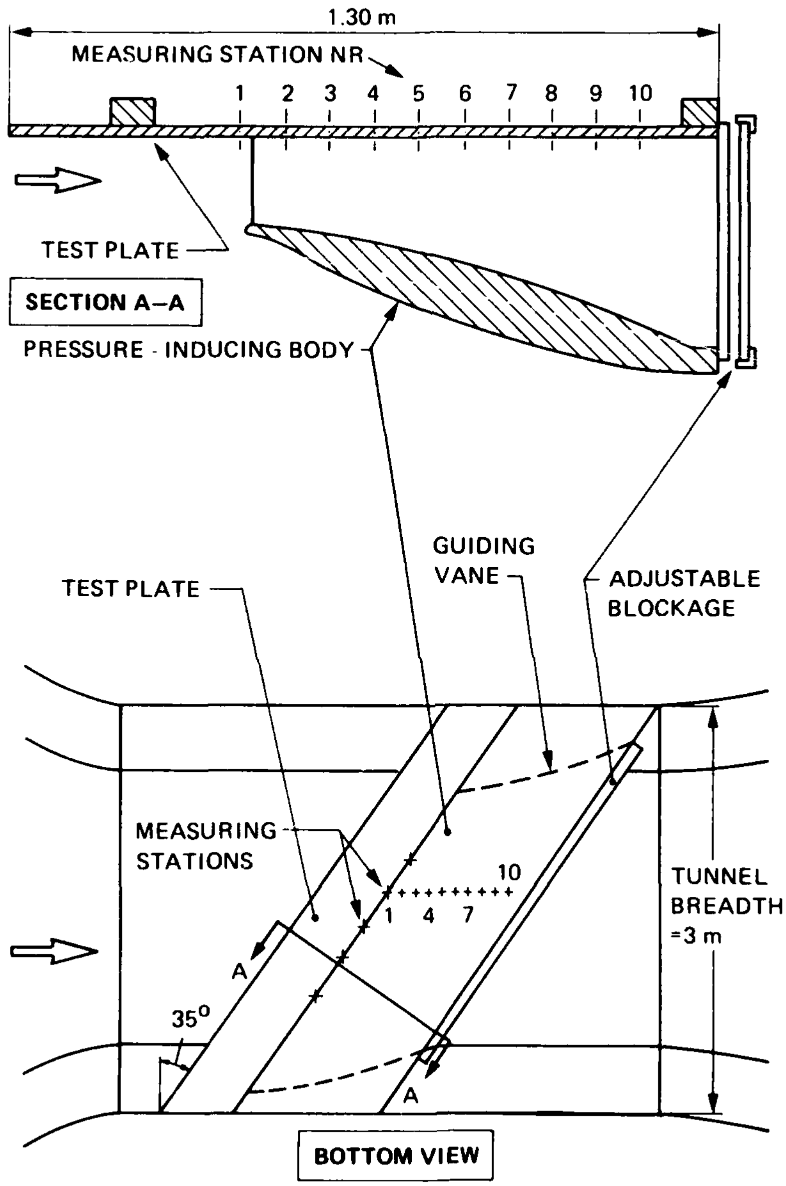

Fig. 1 Sketch of the model used for the NLR experiment

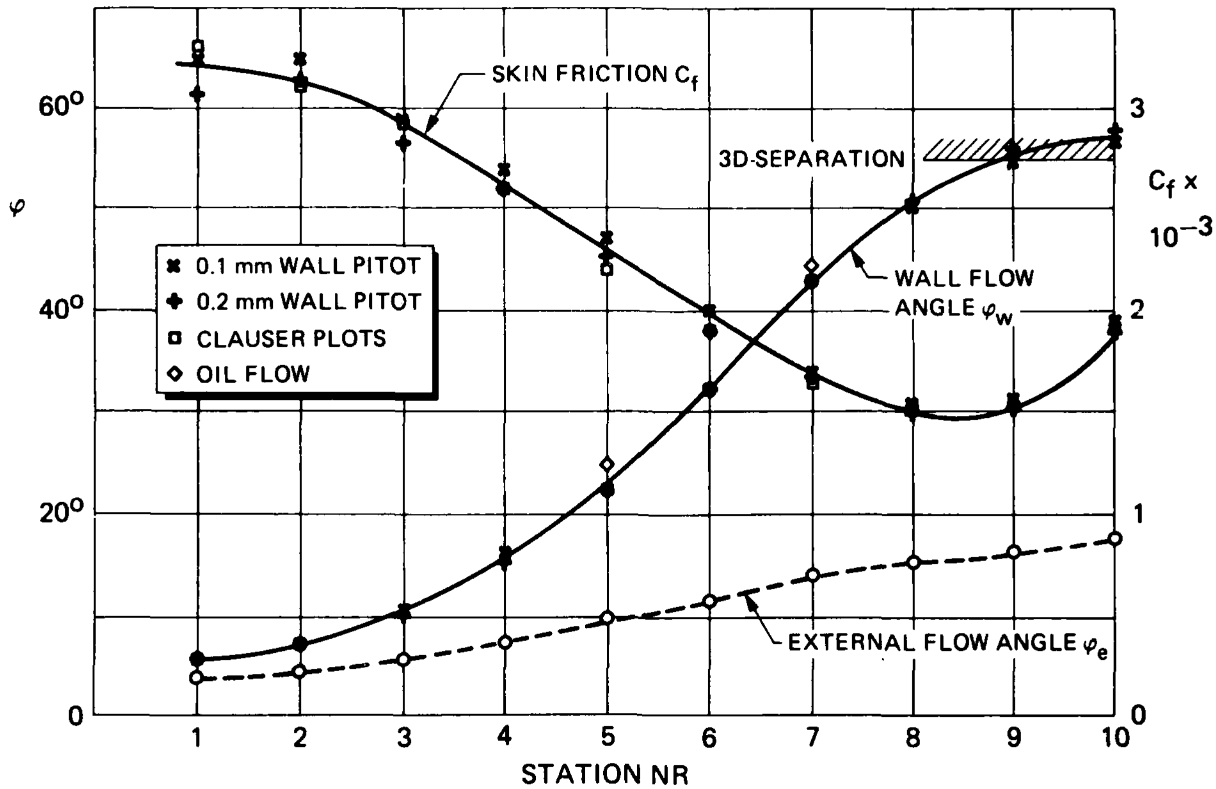

Fig. 2 Measured variation of the external and wall flow angle and the skin friction in the NLR experiment

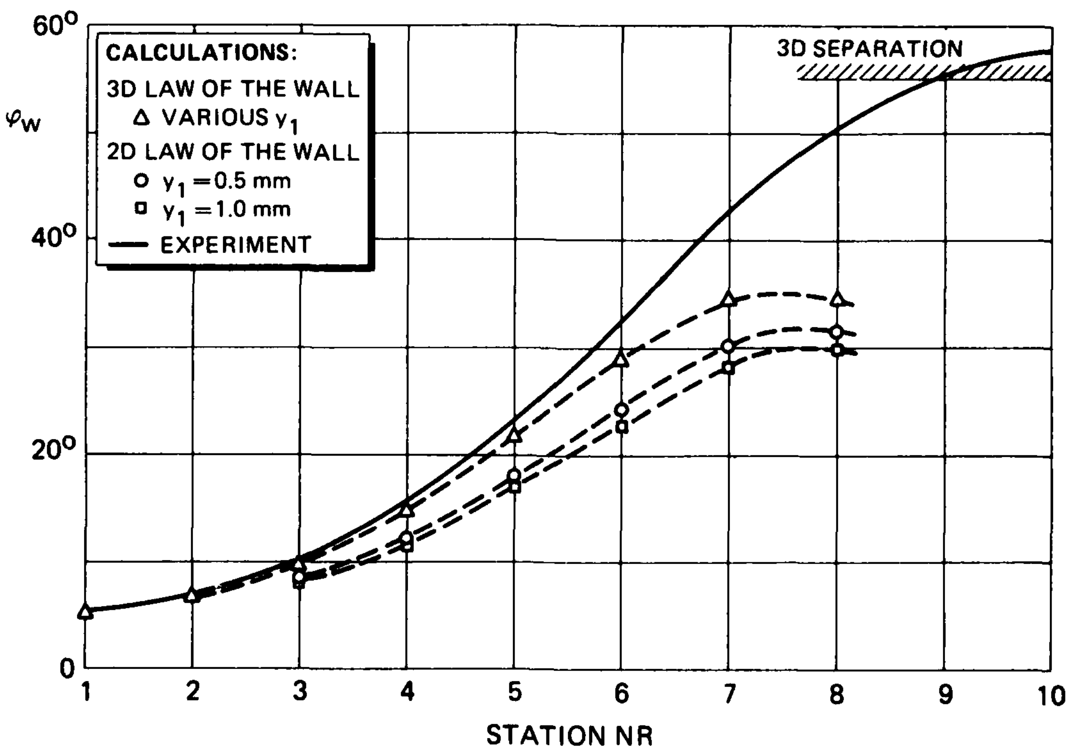

Fig. 3 Measured variation of the wall flow angle and calculation results using Bradshaw's turbulence model

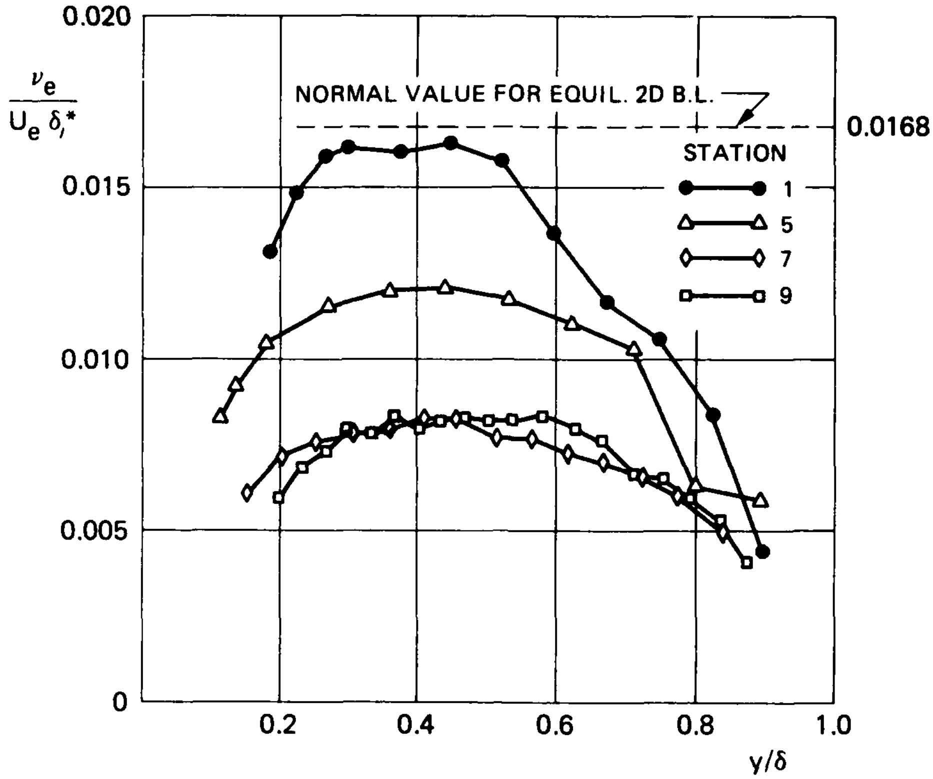

Fig. 4 Eddy viscosity according to turbulence measurements

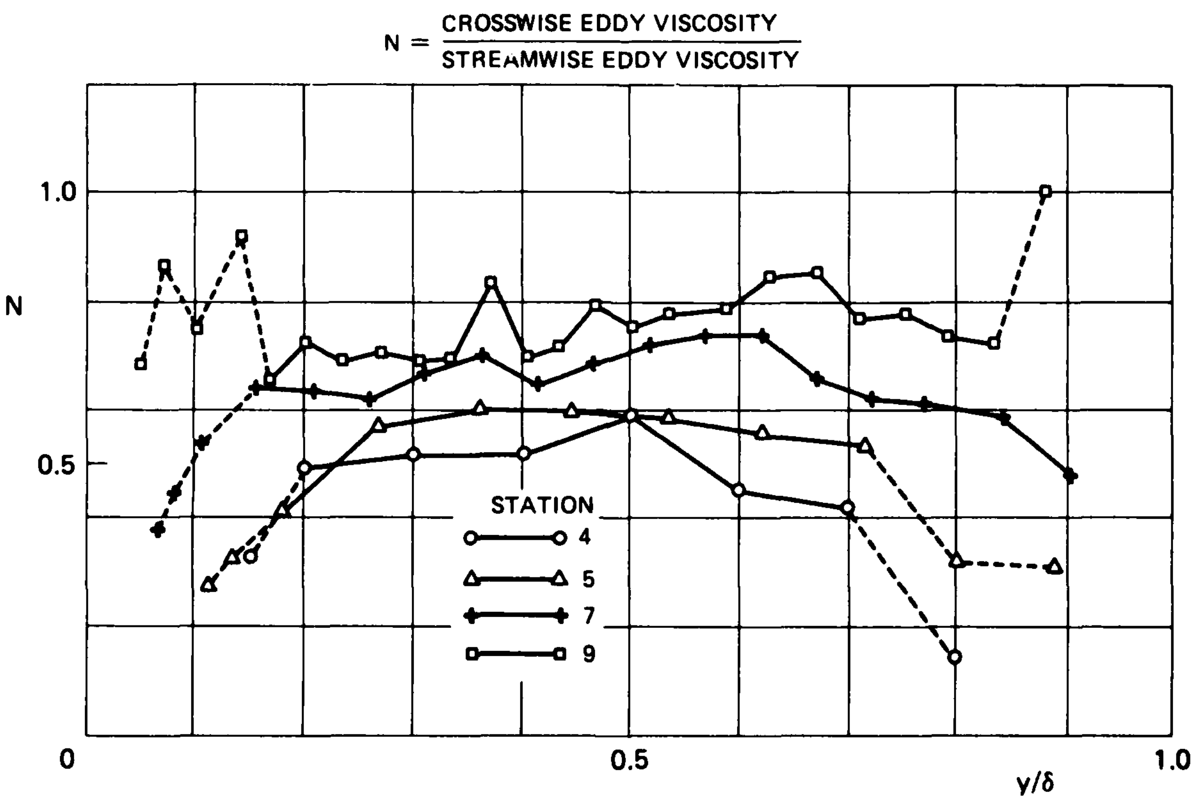

Fig. 5 Non-isotropy of the eddy viscosity according to turbulence measurements

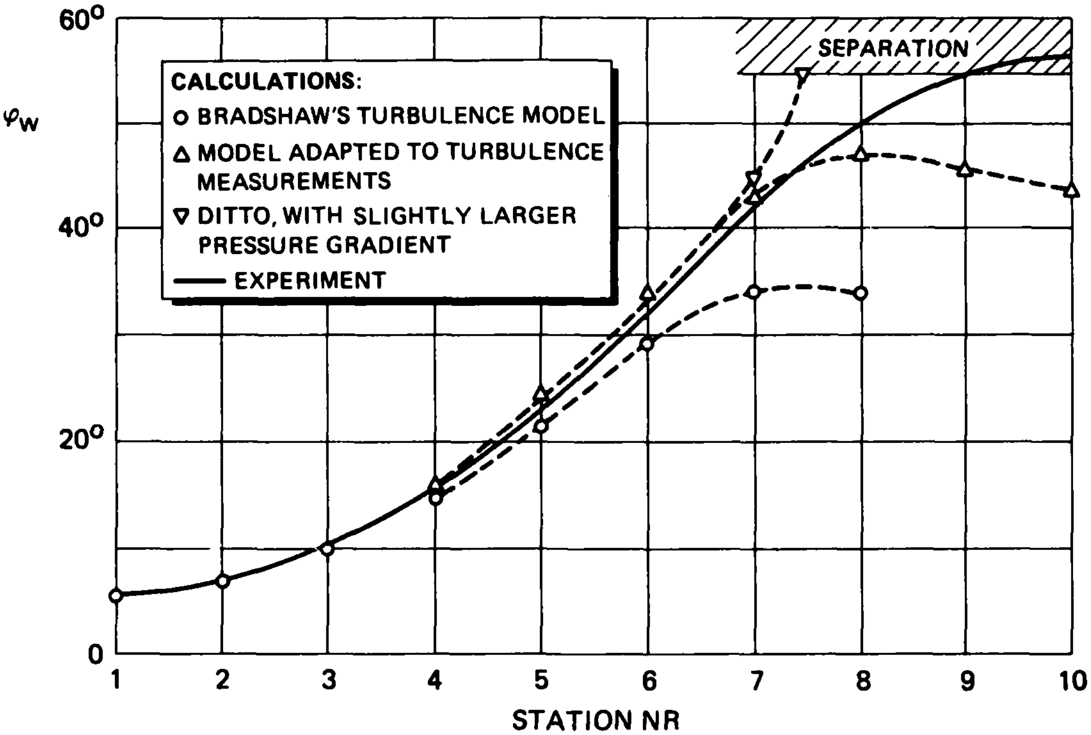

Fig. 6 Calculation results with different turbulence models

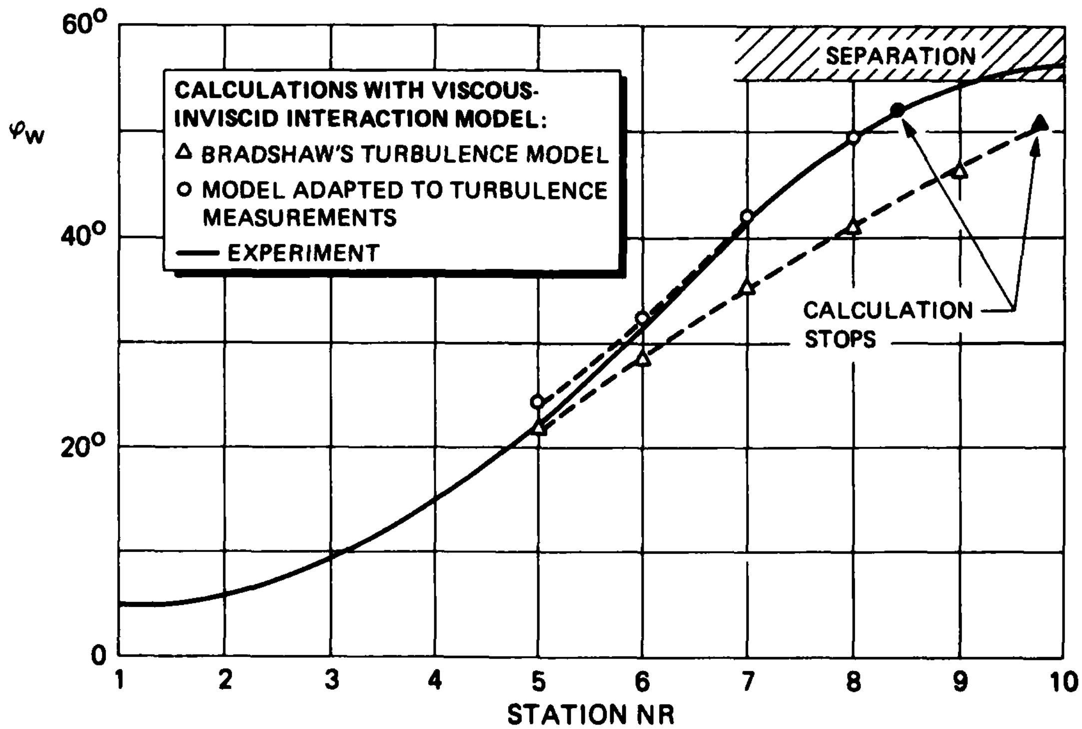

Fig. 7 Calculation results using a simple viscous-inviscid interaction model

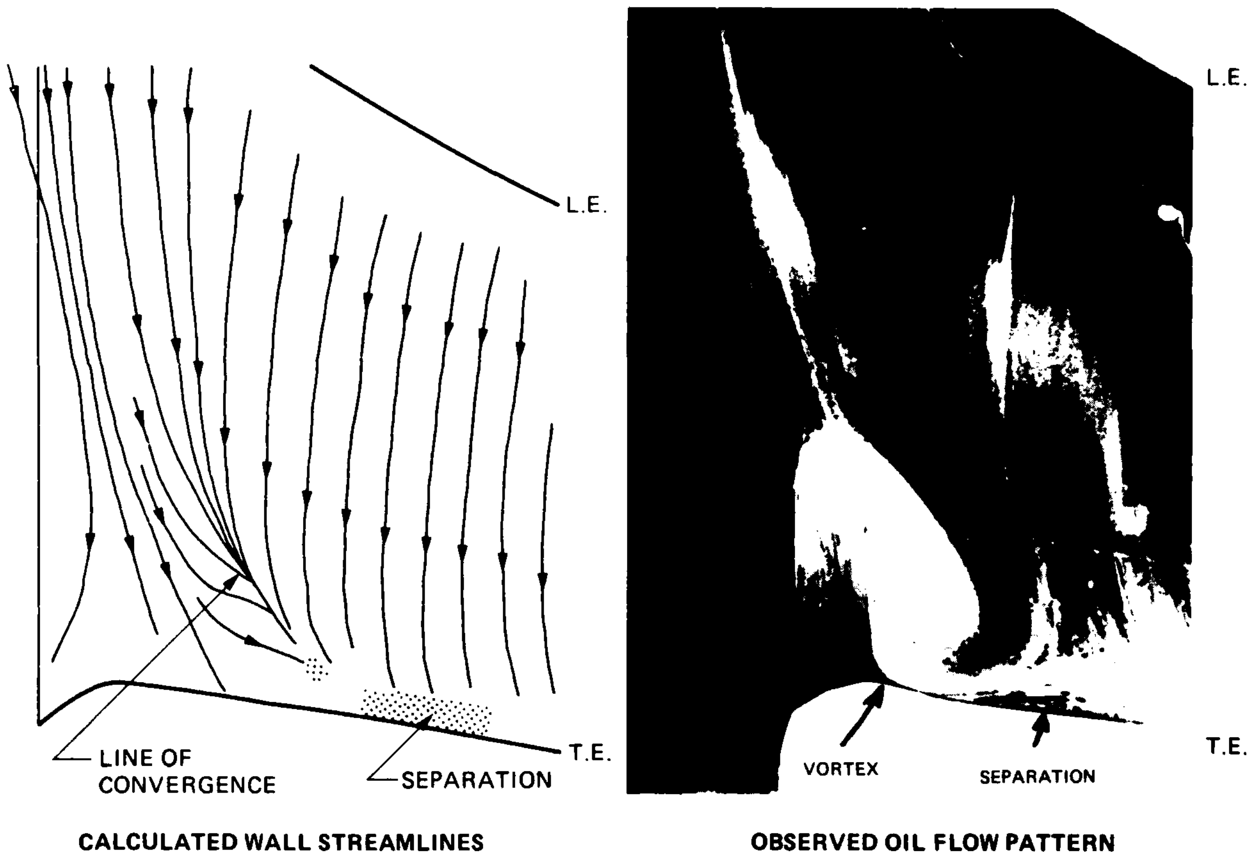

Fig. 8 Results of practical calculations on the root section of a swept wing

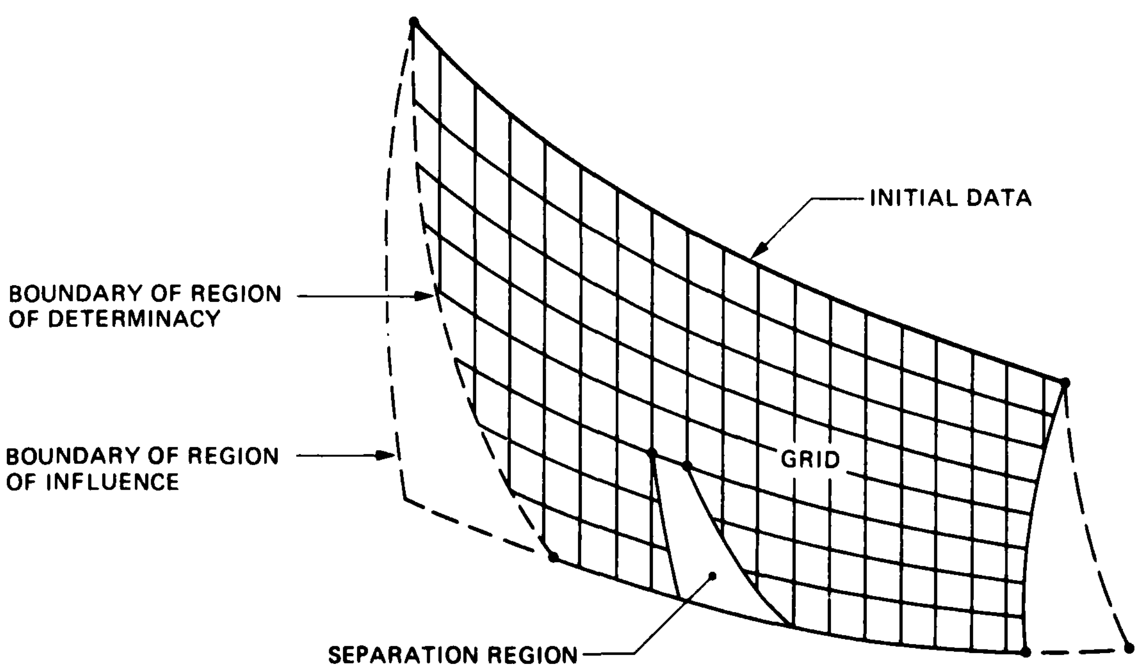

Fig. 9 Typical computational domain of a three-dimensional boundary layer calculation

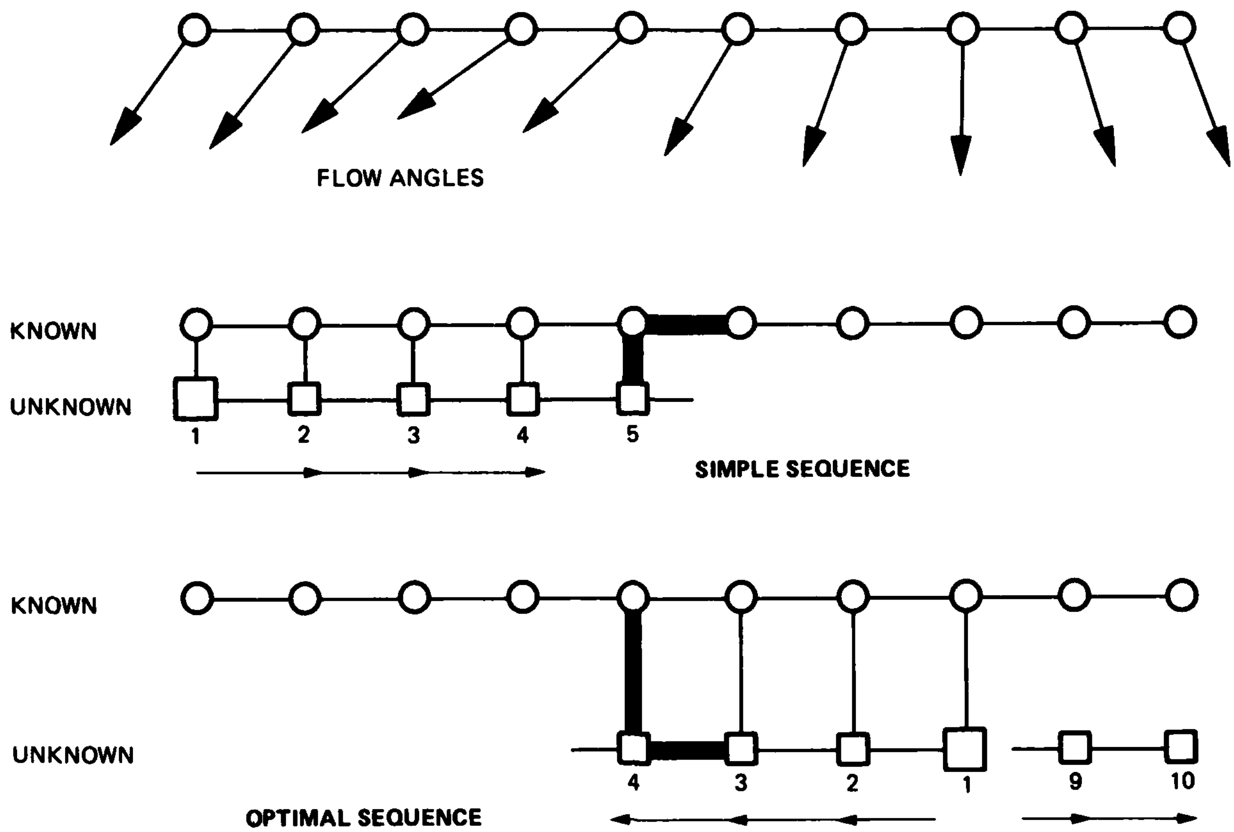

Fig. 10 Scanning sequence of the computations in the new calculation method

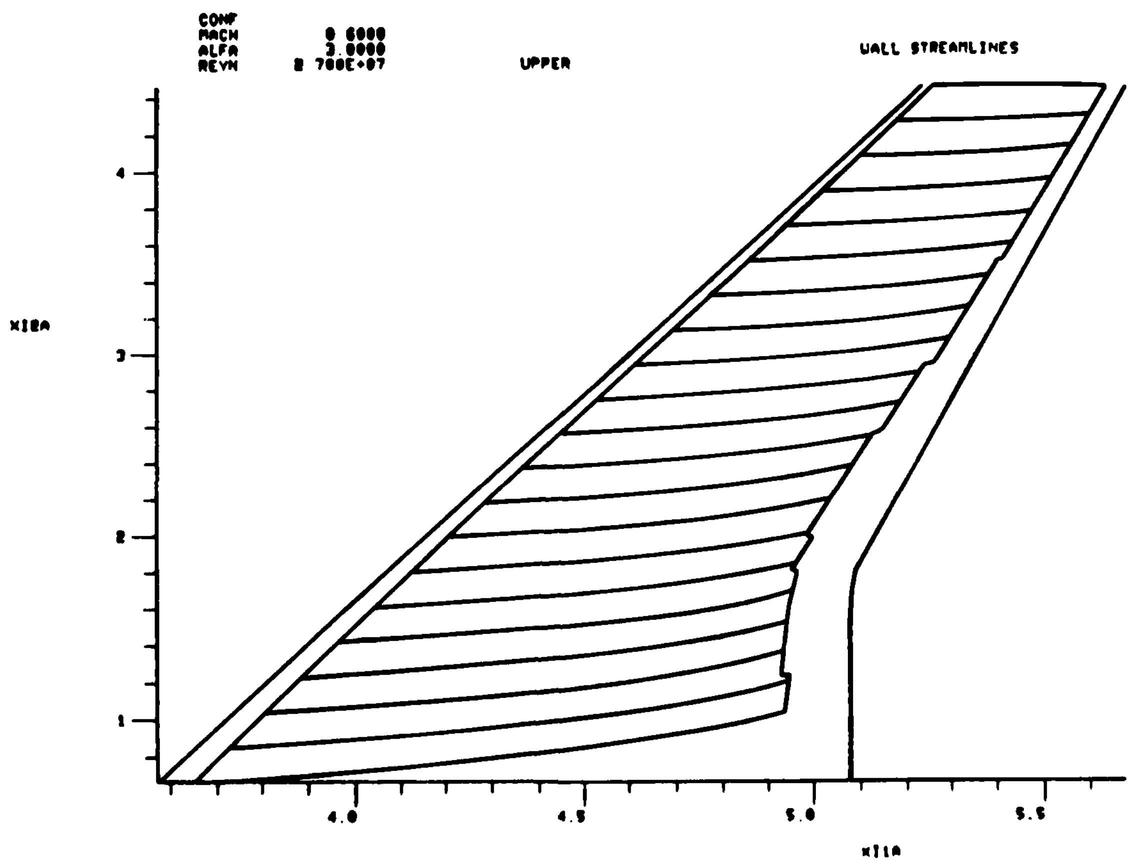

Fig. 11 Standard graphics output of a viscous-inviscid calculation system, showing the wall streamlines on a swept wing

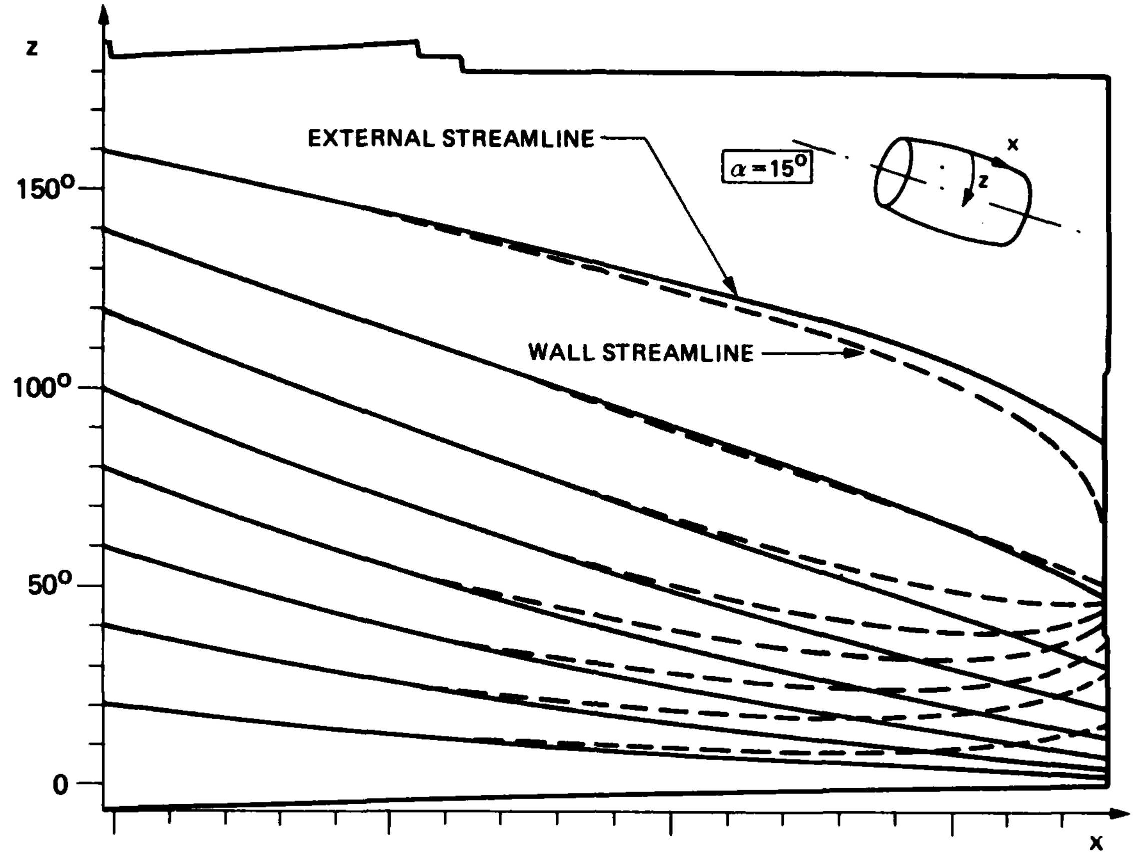

Fig. 12 Calculated external and wall streamlines on an engine cowl at angle of attack

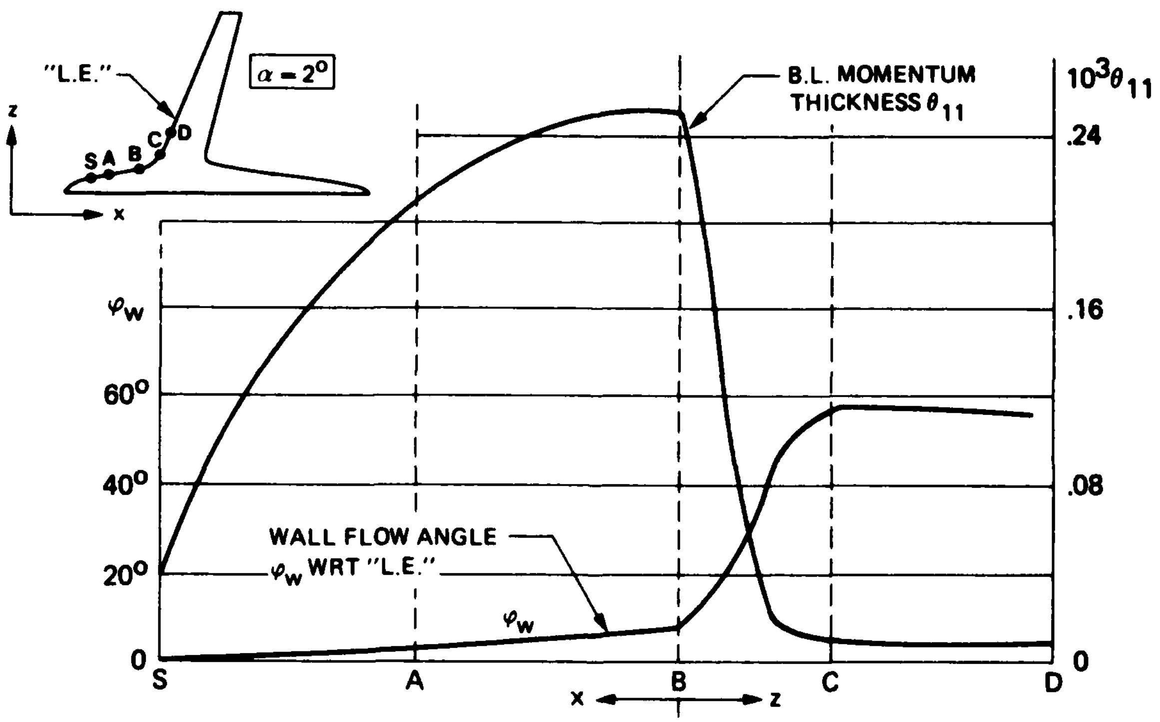

Fig. 13 Boundary layer on a wing-body configuration: calculation results along the "leading edge"

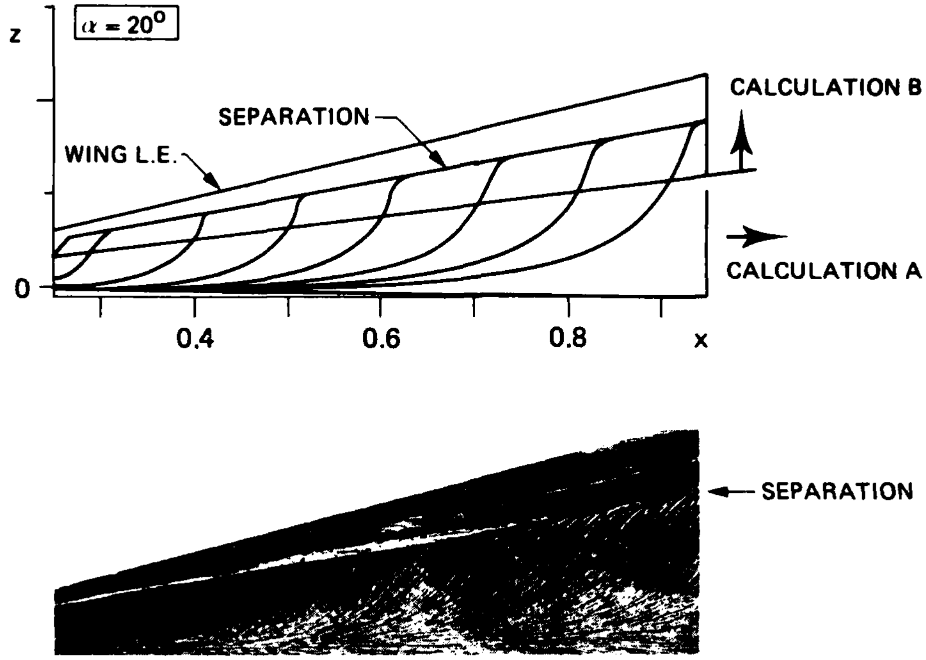

Fig. 14 Calculated wall streamlines and observed oil flow pattern on the leeward surface of a delta wing

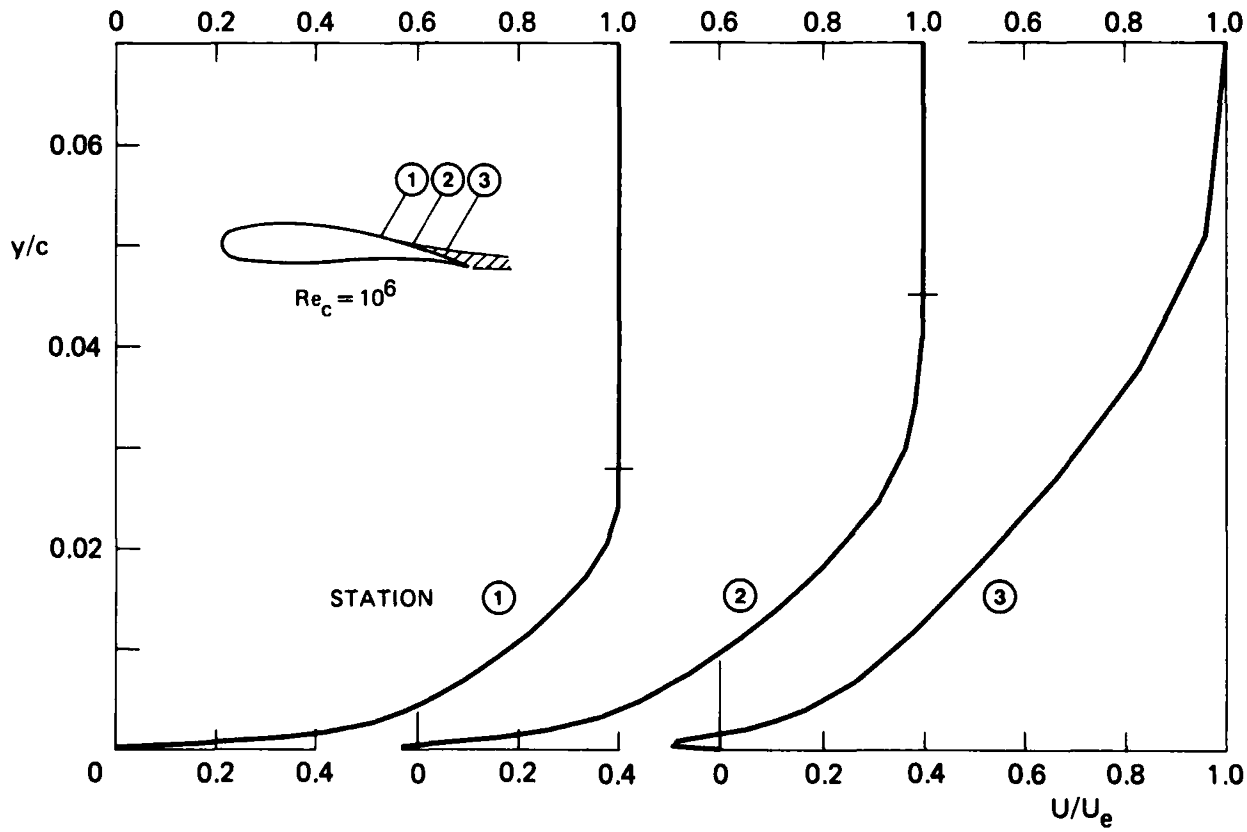

Fig. 15 Some preliminary calculation results of a separating turbulent boundary layer, using a strong interaction approach

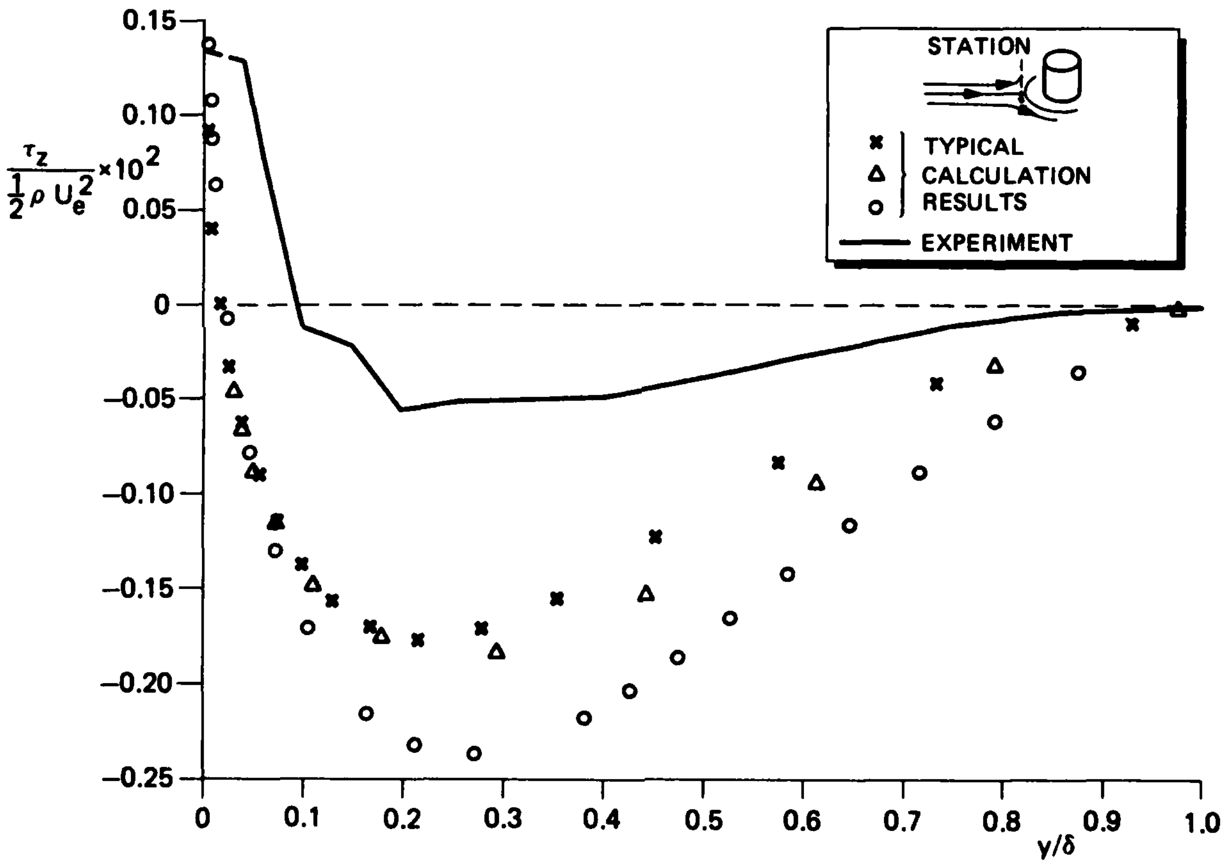

Fig. 16 Comparison of the measured and calculated cross-flow shear stress component: typical results of the 1982 Berlin Workshop

BRIEF REVIEW OF CURRENT WORK IN THE U. K. ON THREE DIMENSIONAL BOUNDARY LAYERS

by

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SUMMARY

The activities summarised cover work in progress in industry, the universities and the R.A.E. They include theoretical work on laminar boundary layers (flow round obstacles, outside streamwise corners, internal flows in turbo-machinery), transition effects on bodies at large angles of incidence associated with cross flow and leading edge contamination, experimental and theoretical work on turbulent boundary layers (swept wings with and without separation, curvature and cross flow, blunt trailing edges, wakes with large cross flow, flow outside streamwise corners, integral and finite difference prediction methods both direct and inverse).

It is increasingly appreciated that more work is needed on the fundamental effects of cross flow on the dynamics of turbulence.

1. INTRODUCTION

In the following a brief review is presented of the current work in the U. K. on three dimensional boundary layers. Such work is in progress at B.Ae. (Brough), at R.A.E. and at eight Universities and Colleges /Cambridge, Cranfield, Imperial College (London), Manchester, Queen Mary College (London), Salford, Southampton, U.M.I.S.T./

These activities do not readily lend themselves to a unified presentation but they include both experimental and theoretical work and they range over laminar flows (F.T. Smith, I.C.; Patel, Q.M.C.; Launder, U.M.I.S.T.), transitional flows (Poll, Cranfield; Lamont, Manchester), and turbulent flows (Cross, B.Ae.; Bradshaw et al.,I.C.; Young et al., Q.M.C.; Sutton and Squire, Cambridge; Stollery, Cranfield; P.D. Smith and Firmin, R.A.E.; Myring, Salford; Lilley, Southampton; Launder et al., U.M.I.S.T.).

2. BRITISH AEROSPACE (BROUGH)

A.G.T. Cross has concentrated on the development of integral methods for turbulent boundary layers because of their relative economy in computing time and costs for typical wing-body arrangements. He argues that the main weakness of most integral methods lies in the somewhat crude forms of the velocity profiles that have been assumed. He has successfully developed in two dimensions a form of velocity profile combining the law of the wall with a wake law, but the latter includes as a parameter an empirically determined function of the non-dimensional pressure gradient and the degree of departure from equilibrium. This is claimed to permit a better prediction of the flow characteristics where separation is imminent than other integral methods and comparison with a wide range of experimental results supports this claim.[1]

Encouraged by this success he has extended his velocity profile to three dimensions, assuming that the law of the wall component lines up with the limiting streamline but the wake component direction changes with distance from the surface. Relations for determining the direction were obtained empirically. In general the method appears to perform well in comparison with other direct methods[2], as is illustrated by Fig. 17.

Cross is at present developing an inverse form of his method with which he hopes to deal with flows involving limited regions of separation. Initial comparisons with measurements in two-dimensional separating flows are very encouraging.[3]

3. CAMBRIDGE

There are three different relevant experimental investigations in progress or recently completed at Cambridge.

Dr. Sutton and his students are investigating the flow over a swept plate of finite thickness, the leading edge is bluff with salient edges from which separation and reattachment in the form of a bubble or vortex sheet can be expected. Various angles of sweep from 30° - 60° have been investigated and the tests have included pressure distribution measurements and flow visualisation. Some results are reported in Ref. 4. They reveal a remarkable sensitivity to small, almost imperceptable, leading edge imperfections from which leading edge vortices can be triggered with associated significant changes in pressure distribution below the vortices as they trail downstream. These investigations are currently being extended to cover a wider range of sweep angle and to include detailed traverses away from the surface.

Dr. Squire and a student (Lai) have just completed some work on the influence on the boundary layer over the rear of an axi-symmetric body of a jet emerging from the body. The jet changed the pressure distribution over the aft body significantly, increasing the entrainment into the boundary layer and the streamwise velocity component and decreasing the turbulence intensity. The turbulence is in any case considerably reduced as compared with that of a flat plate two dimensional boundary layer because of the combined effects of flow convergence and surface curvature.[5]

Another student (Copley) is engaged in an investigation of the flow over a train in yaw. This work is planned to cover pressure distributions, flow visualisation and traverses of wake vortices, with and without ground effect. It is intended to examine the applicability of current prediction methods to such a flow. The work is still at an early stage but some preliminary measurements have been made.

4. CRANFIELD

4.1. Dr. Poll (Refs. 6-9) has made major investigations of the factors that determine transition from laminar to turbulent flow in three dimensional boundary layers. He has established criteria which enable designers to assess the relative likelihood of transition on a swept wing arising from instabilities of the Tollmien - Schlichting waves of classical two dimensional flow theory, cross flow instability near the leading edge or spanwise contamination from a disturbance source on the leading edge. He has been able to extend his analysis to bodies of revolution at incidence and a comparison of his predictions with the experimental results of Dr. Lamont (Ref. 7, 22, 23, see below) is shown in Fig. 7. The factors that control the flow patterns when flow separation is present are particularly complex, since depending on Reynolds number, incidence and Mach number the boundary layer may separate in a laminar state and then remain separated or reattach in a turbulent condition (short bubble) possibly to separate again later or it may be turbulent ahead of the initial separation which may well occur somewhat upstream of that following a short bubble. Again, small surface imperfections can trigger one pattern of flow rather than another and asymmetry with strong side forces is characteristic of such flows.

An interesting recent development is Poll's model of the intermittency[9] associated with spanwise leading edge contamination based on Emmon's spot theory[10] and Narasimha's application of that theory to the flat plate boundary layer.[11] Poll has extended this work to heat transfer.[12] He is now directing this approach to bluff and missile like configurations at high angles of attack and travelling at high Mach numbers.

4.2. Prof. Stollery and his colleagues are currently working on the glancing interaction of shock waves and boundary layers. Some results are reported in Ref. 13, where it is evident that the interaction process is complicated by the development of corner vortices between the main wall boundary layer and that on the wedge surface generating the shock wave. The planned programme is to include the effects of bluntness and sweep of the shock generator.

5. IMPERIAL COLLEGE

5.1. Prof. Bradshaw and his team have been particularly active in producing experimental data on three dimensional boundary layers, since the inadequacies of prediction methods based solely on two dimensional experiments have become increasingly clear. This has been highlighted by the comparisons of the Van den Berg-Elsenaar results[14] with the predictions of the available theories at the Euromech meeting in Trondheim[15] and subsequently by the comparisons made at the 1980/81 Stanford Conference.[16] Pontikos, a student of Prof. Bradshaw, has made an investigation which in essentials is a repeat of the NLR experiment but the measurements have included not only mean flow and Reynolds stresses but also higher order terms required to help in the assessment of the physical significance of all the terms in the Reynolds stress transport equations.[17] Pontikos's set-up is illustrated in Fig. 1. His results supported the inference from the NLR experiments that the cross flow in a three dimensional boundary layer can act, in some way yet to be explained, to reduce turbulent activity. For example, Fig. 2 shows that the ratio a_1 (shear stress/ 2x turbulent kinetic energy) fell steadily with distance downstream in an adverse gradient and approaching separation whilst Fig. 3 shows similarly the reduction in the transverse transport velocity v_q $(=\overline{vq^2}/\overline{q^2})$, of the turbulent kinetic energy q^2.

A subsequent series of experiments still in progress are basically the same as those described above but with the test surface curved to determine the interaction of surface curvature on the cross flow effects. The rig with the surface convex is illustrated in Fig. 4. Some reduction in turbulence intensity with distance downstream has again been noted but the ratio a_1 does not show any consistent associated reduction, see Fig. 5. This is surprising as some reinforcing of the cross-flow effect by the convex curvature might have been expected. However the results are still being analysed and further experiments on concave curvature need to be done before we can assess in general terms the combined effects of cross flow and curvature.

Another series of experiments in progress involve measurements in the wake of an infinite 35^{o} swept wing with a lower surface boundary layer that is nearly two-dimensional at the trailing edge whilst that on the upper surface has a large cross-flow there (up to about 40^{o}). The aerofoil shape is illustrated in Fig. 6. Measurements include mean flow, Reynolds stresses and other turbulence characteristics by traverses of both boundary layers and their mixing region and the techniques are much the same as in a similar investigation of a two dimensional wake by a former student, Andropoulos.[18] Outside the mixing region the results present no special feature but the complex flow inside the mixing region is such that simple eddy viscosity concepts are inapplicable. This is in contrast to the earlier two-dimensional results and it is evident that the cross flow in three-dimensions is having a crucial effect.

Yet another problem area being investigated by Prof. Bradshaw and his team is the interaction of longitudinal vortices and a turbulent boundary layer. Ref. 19 describes the results of tests involving 1) an isolated vortex in an initially two dimensional boundary layer, 2) a vortex pair in the boundary layer with the flow between moving away from the surface, 3) a vortex pair in the boundary layer with the flow between moving towards the surface. The resulting data cover all the terms in the Reynolds stress transport equations that can be measured directly or reasonably inferred from the measurements. These flows are complex and difficult to summarise briefly. In general the vortices are regions of relatively low turbulence intensity, low primary Reynolds shear stress and low eddy diffusity, whilst a_1 varies markedly but in general is significantly less than in two dimensions. Prof. Bradshaw has concluded that transport equations for the triple products may well be needed in any calculation method to be applied to imbedded vortex flows.

5.2. F.T. Smith has devoted considerable attention to theoretical studies involving three dimensional laminar boundary layers. In Ref. 20 he applied triple deck theory to a two dimensional boundary layer encountering a shallow three dimensional hump. The theory was linearised in terms of the perturbation and did not involve separation. It vividly demonstrated, however, the relatively large effect of the pressure field about the hump on the lowest (viscous) layer of the boundary layer, producing strong secondary flows which reversed with the reversal of the lateral pressure gradient as the flow passed from the front of the hump to the rear. In a later paper (Ref. 27) he applied a non-linear theory to a steeper form of obstacle in the form of a ring-like bump cum trough in a pipe. He found that one part of the flow upstream terminated in a regular separation whilst elsewhere the flow was attached. However, the analysis suggested that the attached flow terminated at a line singularity whilst the separated flow developed as a vortex sheet. It is to be noted that he defined separation as occurring when the axial skin friction component is zero.

F.T. Smith has also done some work on the boundary layer in wing-body junctions on which it is understood a paper should be appearing shortly.

6. UNIVERSITY OF MANCHESTER

Dr. Lamont has made a very detailed experimental study of the flow over inclined bodies of revolution covering a wide range of incidences up to 90^{o} and a range of Reynolds number (Ref. 22,23). In particular, the effects of roll angle were carefully investigated and the results demonstrated the importance of small imperfections in triggering asymmetric transition locations with subsequent large asymmetries in flow pattern. This work was done during sabbatical leave at NASA Ames Research Center and was the subject of the analysis by Dr. Poll of Cranfield referred to above (see Fig. 7). Further joint work on this topic is in progress.

7. QUEEN MARY COLLEGE

The experimental work at Queen Mary College on three dimensional boundary layers has been mainly directed at a) separating boundary layers b) boundary layers inside streamwise corners c) boundary layers outside streamwise corners.

The work on separating boundary layers has been briefly and in part reported in Ref. 24. Two kinds of separating flow have been investigated namely that to be expected on a swept wing of large aspect ratio, i.e. quasi-two dimensional, and that in the region of a wing body junction where the body boundary layer separates ahead of and around the wing leading edge. For the former tests the measurements covered mean flow and turbulence quantities but for the latter only the mean flow quantities were available to be reported on in Ref. 24. However, more recently results obtained by a student, Rios-Chiquete, have become available for the turbulence quantities for the latter investigation.

The main points noted in Ref. 24 were:-

1) There was a strong axial flow in the main reversed flow vortex after separation with axial velocities of the order of the main stream velocity quite close to the surface within the lowest tenth of the boundary layer.

2) The three dimensional separated flow was in general somewhat steadier and grew less rapidly than in two dimensions.

3) The strong axial flow in the reversed flow vortex for the swept wing arrangement was associated with a relatively large turbulence component but not with a correspondingly high shear stress component.

4) The corresponding ratio a_1 showed erratic fluctuations in the separated flow region but was generally small (< 0.1) in the lower half of the boundary layer.

Some of the measured mean flow and turbulence quantities for the wing-body arrangement, due to Rios-Chiquete, are illustrated in Fig. 8, 9, 10.+ As in the earlier work these show evidence of a small counter flowing vortex between two main reverse flow vortices (Fig. 8) and of large axial flow velocities close to the surface in the separated flow region (Fig. 9). Also the large transverse turbulence component is in accordance with point (3) above, as is the small shear stress $\overline{v w}$ (Fig. 10). However, the stress $\overline{u w}$ is rather large suggesting some correlation with the relatively large mean shear dW/dx. Nevertheless, no simple algebraic relation between the eddy stresses and the mean velocity gradients is evident from the data.

The work on the flow inside streamwise corncers of both laminar and turbulent boundary layers has been reported in Ref. 25, 26 and need not be discussed further here.

Work is currently in progress on the flow external to a 90^0 streamwise corner, this work is being done by a student Patel. A theoretical analysis has been developed by Ridha[27] (University College) for the case of laminar flow in a zero streamwise pressure gradient. Patel has investigated the corner with both a profiled and a sharp nose, and for the latter (as in the case of an internal corner) an incidence setting had to be adopted to avoid the introduction of major disturbances at the leading edge due to fluctuating separation bubbles that otherwise occur there. Then, to avoid blockage of the flow inside the corner an upward flap setting at the rear of the model was necessary. In consequence a non-uniform streamwise pressure distribution had to be accepted for the sharp nose corner (Fig. 11). Some non-dimensional velocity profiles are illustrated in Fig. 12 and it will be seen that a comforting degree of similarity was present for the profiled nose but not for the sharp nose. Transverse velocity components, streamwise isovels and skin friction distributions are illustrated in Fig. 13, 14 where comparisons with Ridha's theoretical predictions are indicated. Significant differences between theory and experiment are evident, as in the case of the laminar flow internal to a corner these may be due to modifications introduced by the nose shape.

With the boundary layer turbulent smooth flow for the sharp nosed corner was achieved with a favourable pressure distribution (no flap setting was needed) (Fig. 11). Typical isovels are shown in Fig. 15 which shows the characteristic thinning of the boundary layer on the centre line but also with evidence of secondary vortex formations close by. Reynolds stress distributions are illustrated in Fig. 16. These results are still the subject of analysis.

8. R.A.E.

P.D. Smith is a leading figure at the R.A.E. in the field of three dimensional turbulent boundary layers and his integral method modified to include lag entrainment is well known and widely used (Ref. 28, 29). Recently he has developed an inverse form of his integral method which is giving good results as is illustrated in Fig. 17. This presents the predictions of his inverse method along with those of some other methods and compares them with the experimental results of Elsenaar and Berg for a swept wing.

In the meanwhile his colleague Caroline Boyd is developing a finite difference scheme which promises advantages over other finite difference methods. Details are not yet available but a publication is in preparation and further work on the finite difference scheme is also being done at Southampton University (see below).

At R.A.E., Bedford, a large skin friction meter has been built for testing various surfaces designed to modify the large eddy structure in turbulent boundary layers. Work on a yawed step is planned involving triple hot wires.

At R.A.E., Farnborough, experimental work by Firmin has been in progress on the boundary layer characteristics for two wing sections (R.A.E. 101 and 2822) with and without blunt trailing edges over a wide incidence range with sweep of 0^0 and about 28^0. Work in the 8' x 6' tunnel is also planned on the boundary layers on a fighter type wing of aspect ratio about 3.8 with the object of providing a severe but practical test of current prediction methods.

9. SALFORD

Dr. Myring has concentrated his attention in recent years on the development of prediction methods for internal flows in ducts using an interaction scheme between the boundary layer and an inviscid core flow. The emphasis has been on the use of general non-orthogonal curvilinear coordinates and his methods involve a three dimensional grid generator scheme coupled with an Euler code capable of dealing with core vorticity and total temperature gradients. Integral as well as differential methods are being used for the boundary layer flow; the turbulence models are not new but zero, one and two equation models are being used. The present work has been limited to subsonic flows but work has started on transonic and supersonic flows. Publication of some of this work is now being considered.

+ The notation used is illustrated in Fig. 8.

10. SOUTHAMPTON UNIVERSITY

Prof. Lilley is currently heavily involved with the planning and organisation of a successor conference to the 1980-81 Stanford Conference on complex turbulent flows.[16] This next conference is to be held in Southampton in 1985. As noted above he also has a research contract with the R.A.E., to assist in the development of the finite difference scheme for three dimensional turbulent boundary layers initiated by Caroline Boyd. The results given by this method and P.D. Smith's integral method appear to differ in certain cases and the explanation appears to lie in the turbulence model of the former method. Prof. Lilley plans to explore this question in detail and will endeavour to improve the turbulence model. He hopes to be guided b he physical ideas emerging from his recent work on coherent structures in turbulent flow (Ref. 30).

He is also hoping to embark on an experimental programme on three dimensional turbulent boundary layers with large angular changes in the mean velocity vector with distance normal to the wall.

11. UNIVERSITY OF MANCHESTER INSTITUTE OF SCIENCE AND TECHNOLOGY (UMIST)

Prof. Launder and his colleaques have made major contributions to the study of internal flows in turbo machinery and other relevant industrial applications.

For these problems the special features are:-

1) The Reynolds numbers are sometimes low compared with those of external aerodynamics.

2) The distinction, between flow regions which can be treated as viscous or inviscid is often difficult to define.

3) Temperature and buoyancy effects as well as heat transfer are frequently of major importance as are the mixing processes of different fluids or different phases.

Prof. Launder has been a very active worker in the field of turbulence modelling, his current interests are tending to focus on algebraic stress models.

Particular problems on which he and his colleagues are currently engaged are discrete-hole cooling processes (Ref. 31), the dispersal of pollutants in a turbulent boundary layer (Ref. 32), flow in a spirally fluted tube (Ref. 33 describes some laminar flow calculations, turbulent flow calculations are planned), flow round bends (Ref. 34 describes some numerical work aimed at calculating the laminar flow round a 180° toroidal bend, work on turbulent flow is planned). This work is in collaboration with Prof. J.A.C. Humphrey of the University of California. Ref. 35 also contains a useful review of current knowledge on turbulent wall jets with a helpful critique of existing turbulence models.

12. CONCLUDING REMARKS

From the foregoing it is evident that there is work of significance on most of the major aspects of three dimensional boundary layers in progress in the U.K. Should the F.D.P. decide to hold a Symposium on this topic then it seems likely that a substantial input of papers can be expected from the U.K.

It is now evident that three dimensional boundary layers are in general sufficiently different from two dimensional boundary layers not to be readily predicted by methods that are simple extensions of the latter. Therefore, experimenters are increasingly concentrating on the physical effects of cross flows and the associated mean shears. These seem likely to be as important as longitudinal curvature was shown to be by Bradshaw[36] more than a decade ago.

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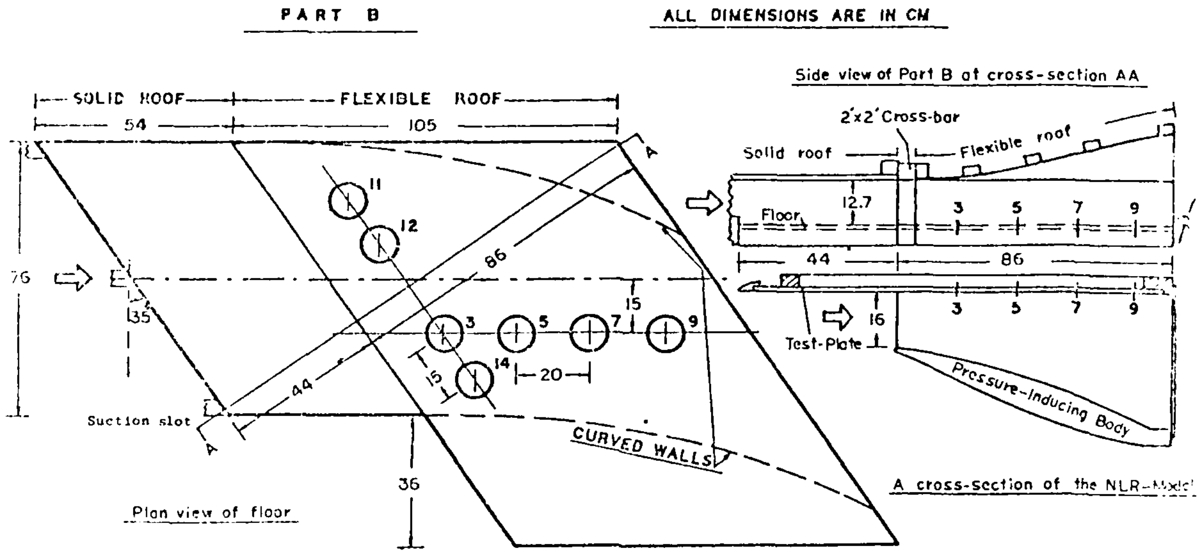


Fig. 1 PLAN AND CROSS SECTION DIMENSIONS OF TEST SECTION PART B, (Bradshaw & Pontikos, Ref. 17) INCLUDING THE CROSS SECTION OF THE NLR MODEL FOR COMPARISON.

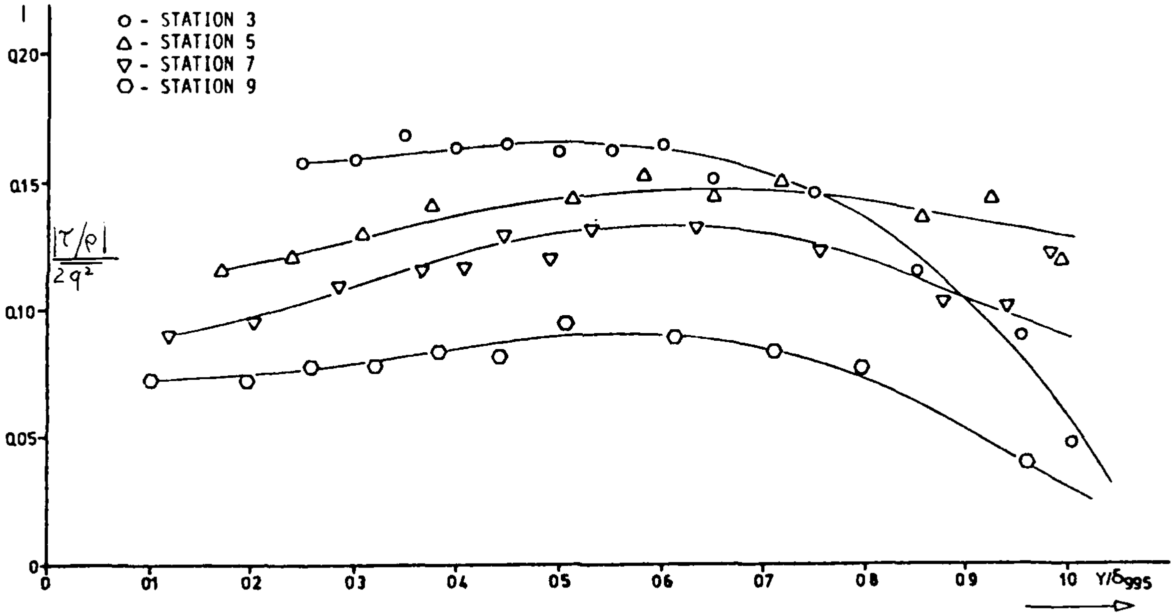


Fig. 2 The ratio of shear stress to turbulent kinetic energy at the chordwise stations. (BRADSHAW & PONTIKOS)

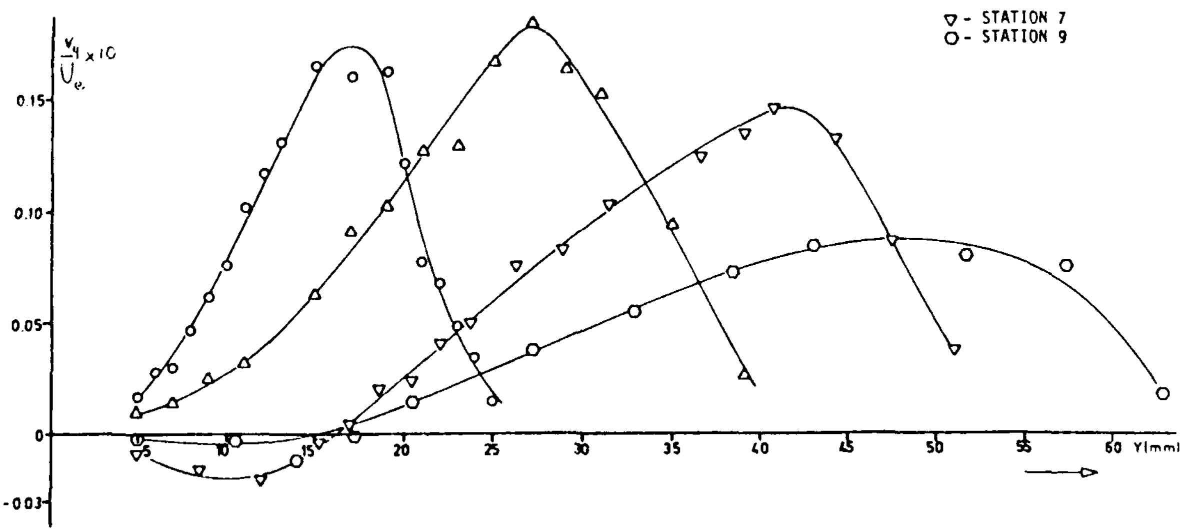
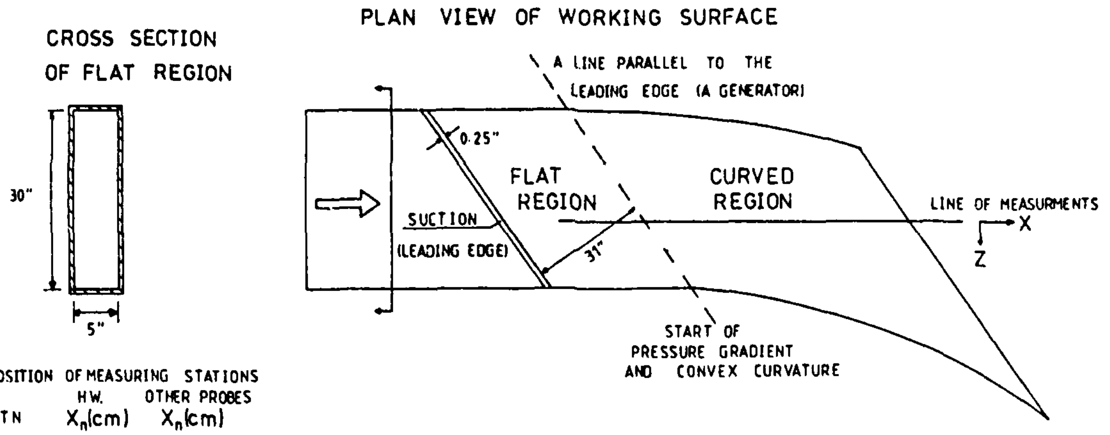


Fig. 3 The v_q turbulent kinetic energy transport velocity at the chordwise stations. (BRADSHAW & PONTIKOS)



POSITION OF MEASURING STATIONS

STN	H.W. X_n(cm)	OTHER PROBES X_n(cm)
2		-40.16
3	-20.30	-27.46
4		-10.31
5		1.27
6	7.97	13.97
7	24.67	26.67
8	41.37	44.45
9	56.15	62.23
10	68.93	74.78
11	85.48	92.56

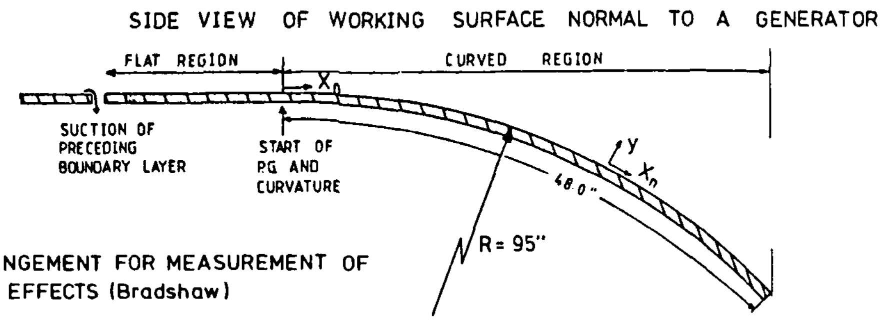


Fig. 4 TEST ARRANGEMENT FOR MEASUREMENT OF CURVATURE EFFECTS (Bradshaw)

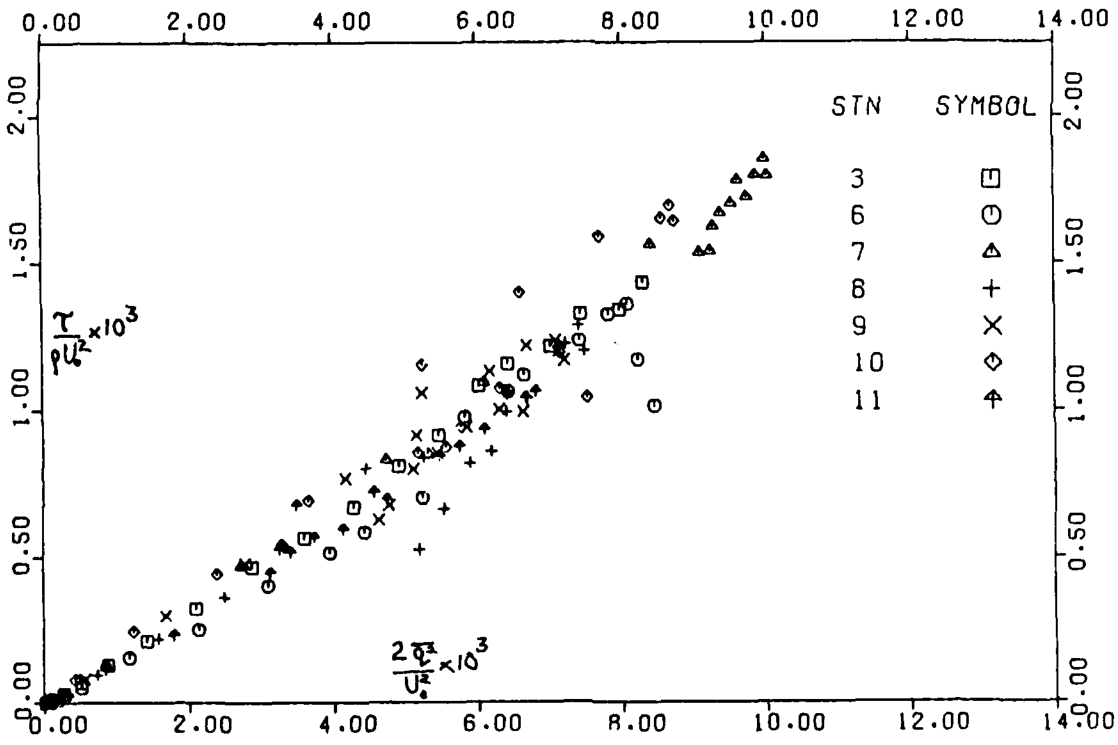


Fig. 5 Resultant shear stress v 2 x turbulent kinetic energy on curved surface (Bradshaw)

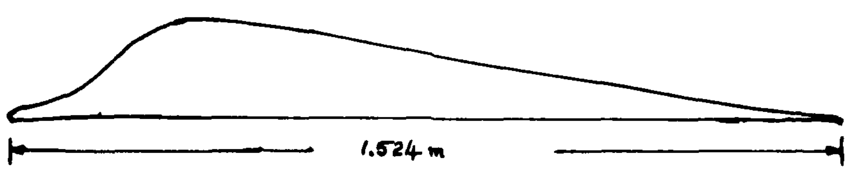
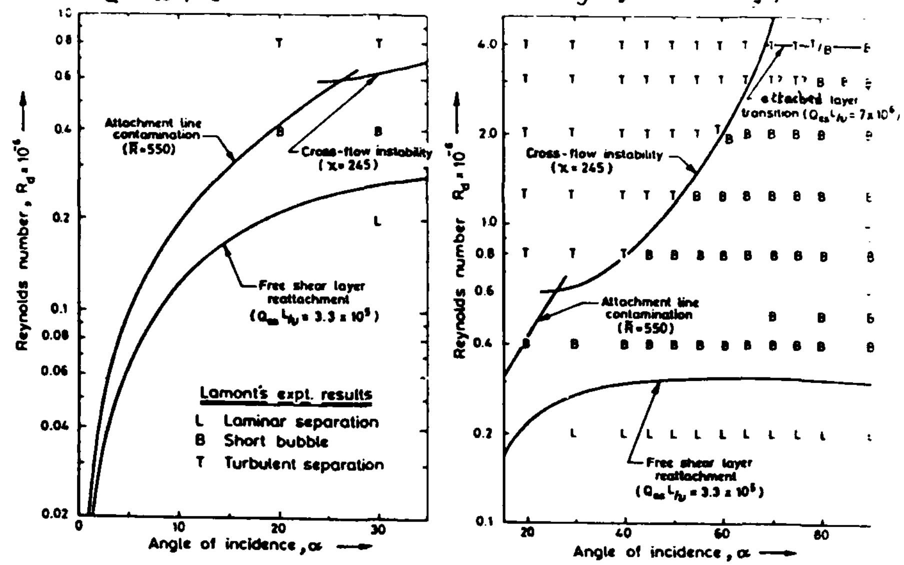


Fig. 6 Aerofoil Section of wake model (Bradshaw)

Q_{es} = result$^{\underline{t}}$ vel^{y} at edge of b.l. at separation, d = body diameter
$R_d = Q_\infty d/\nu$, L = characteristic streamwise length, $\bar{R}$ = similarity parameter



(a) Incidences up to 35°.

(b) Incidences from 20° to 90°

Figure 7 A comparison between the predicted transition boundaries and the boundaries determined experimentally by Lamont. (Poll)

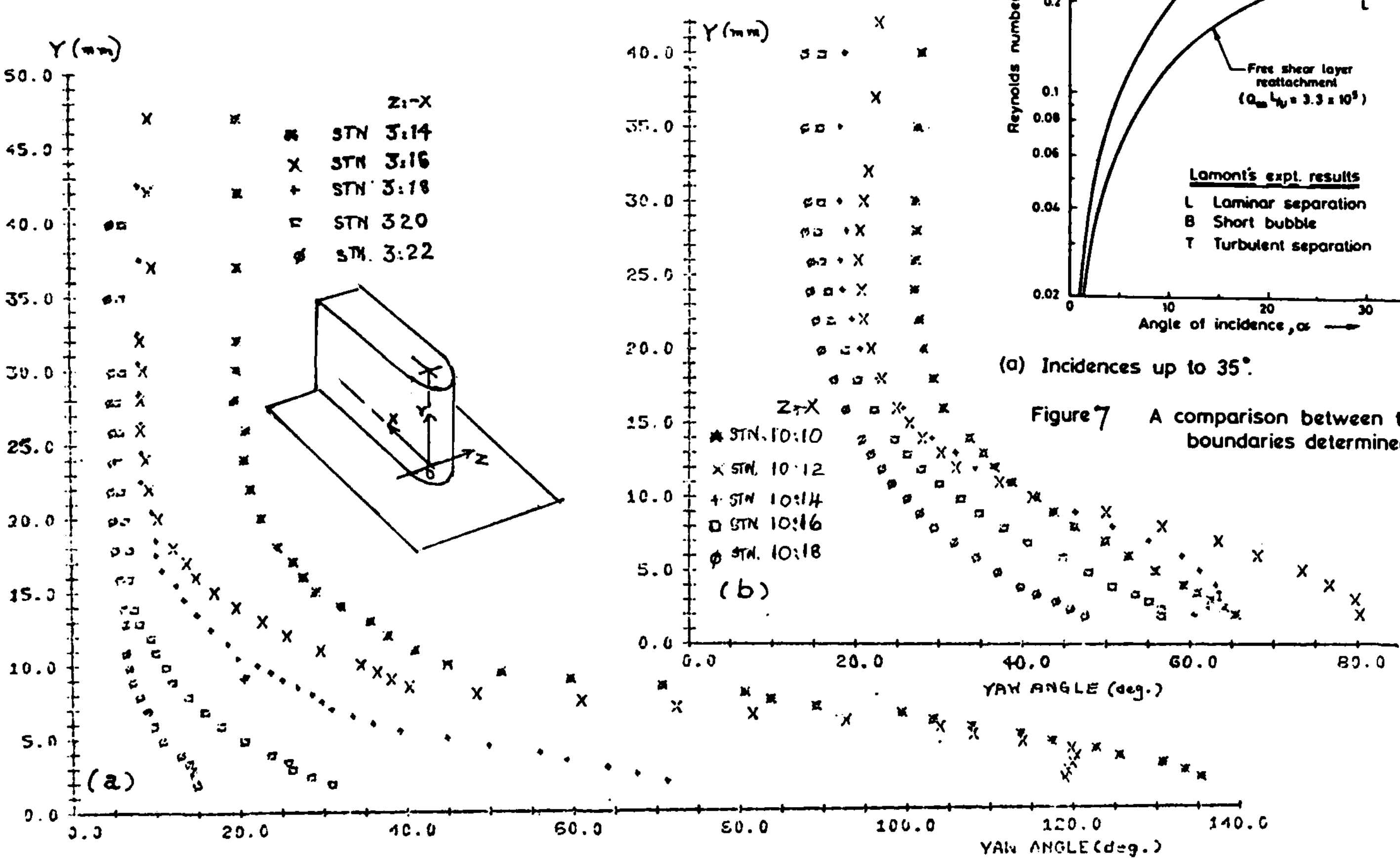


Fig.8 Yaw angle distributions in boundary layer in wing-plate junction. (a) Z = 3. (b) Z = 10
(Rios-Chiquete)

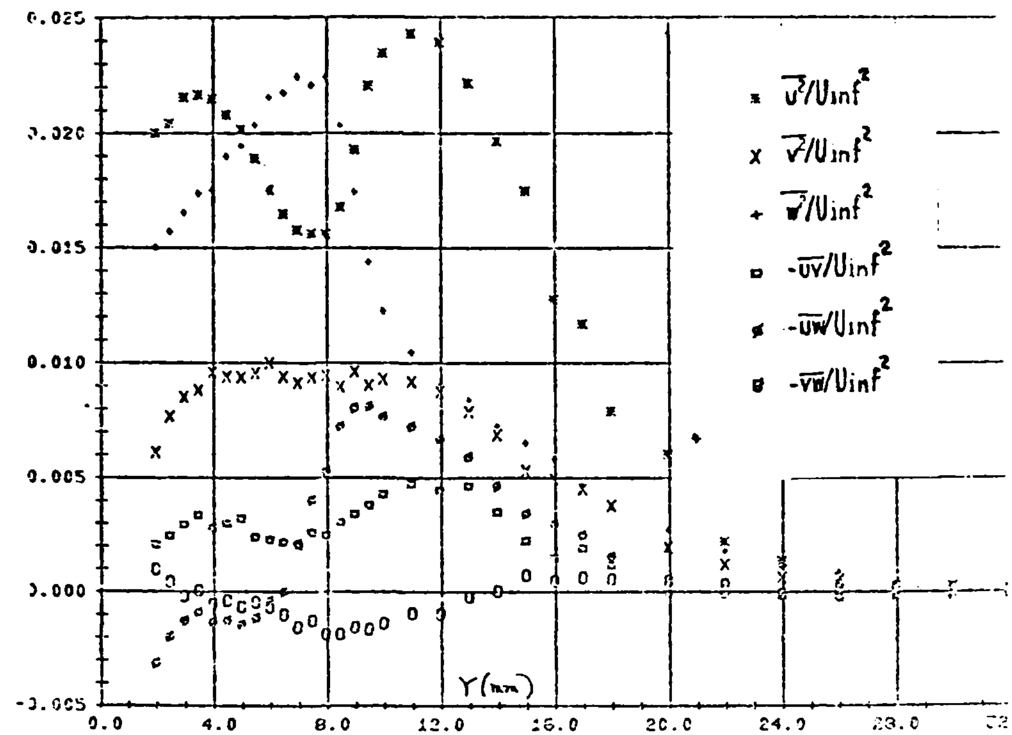


Fig.10 Eddy stress distributions in wing-plate junction, station 3:14 (Rias-Uh)

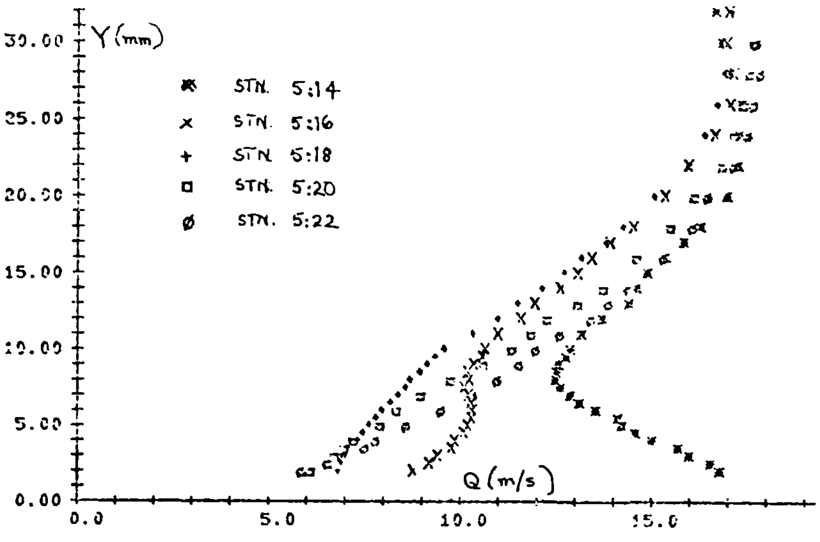
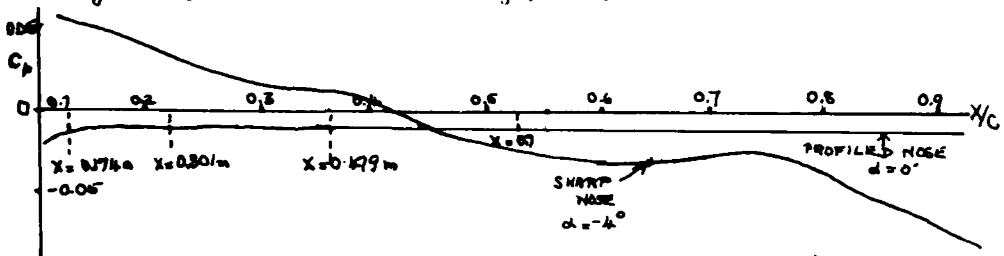
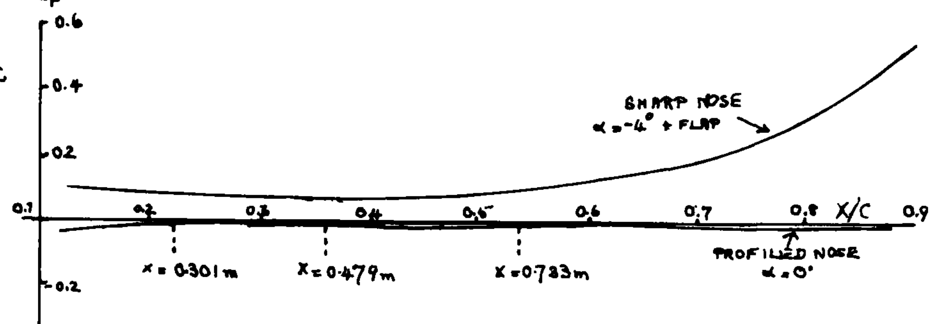


Fig.9 Resultant velocity distributions in wing-plate junction, Z=5



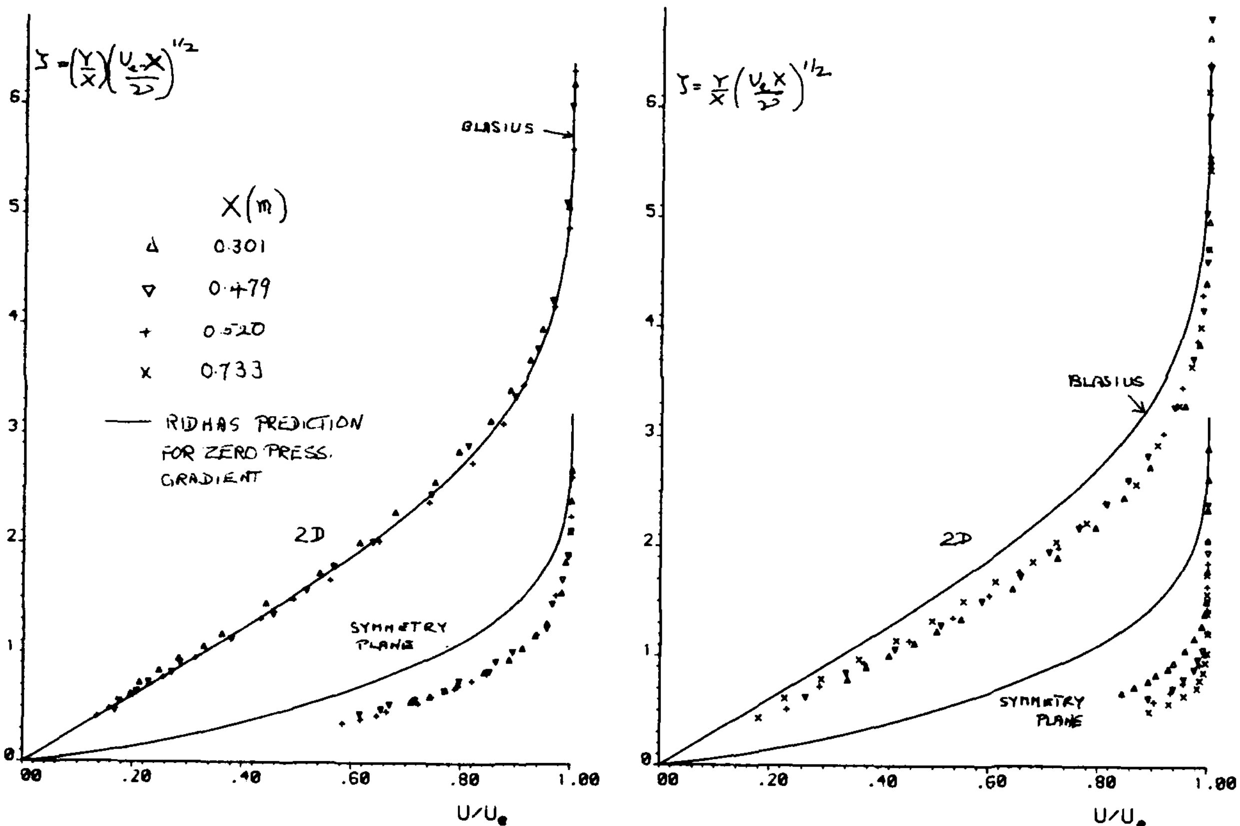
(b) TURBULENT:– U_∞ = 30.0 m/s FOR PROFILED NOSE (α=0°)
U_∞ = 13.3 m/s FOR SHARP NOSE (α=-4°)



STREAMWISE CORNER FLOW PRESSURE DISTRIBUTIONS.
(a) LAMINAR:– U_∞ = 2.75 m/s.

Fig.11 CORNER FLOW PRESSURE DISTRIBUTIONS (PATEL)

FIG. 12
NON-DIMENSIONAL STREAMWISE VELOCITY DISTRIBUTIONS FOR LAMINAR BOUNDARY LAYER EXTERNAL TO STREAMWISE CORNER. $U_\infty = 2.76$ m/s. (PATEL)



(a) PROFILED NOSE (b) SHARP NOSE

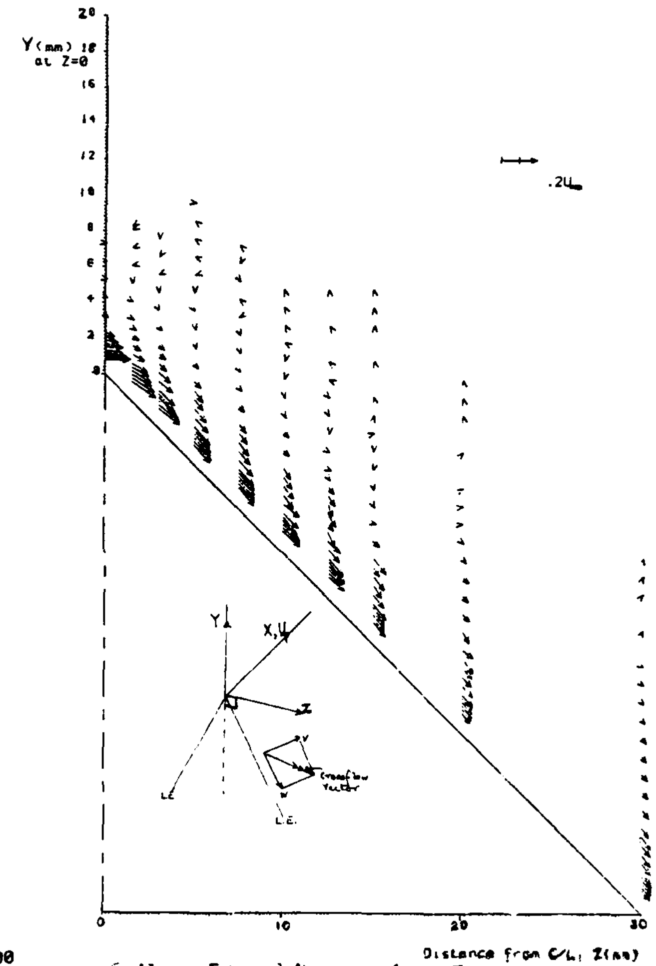


Fig 13 External Streamwise Corner Flow, Laminar
Crossflow Vectors at X = .7328m. and U_∞ = 2.75m/s
(PATEL)

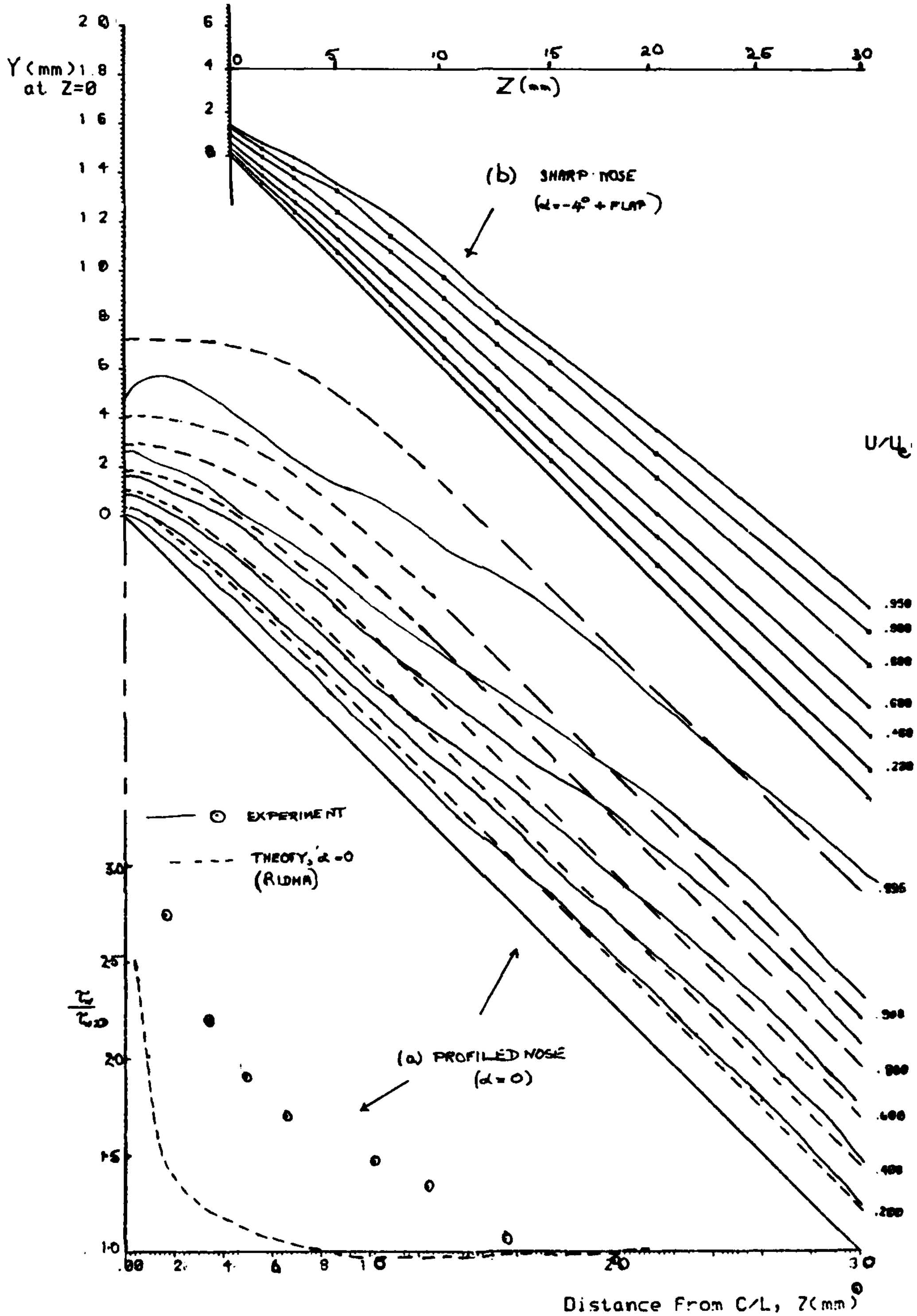


Fig. 14 External Streamwise Corner Flow, Laminar
Meanflow Isovels at X= .7328m. and U_∞= 2.75m/s, and spanwise distribution of skin friction (Patel)

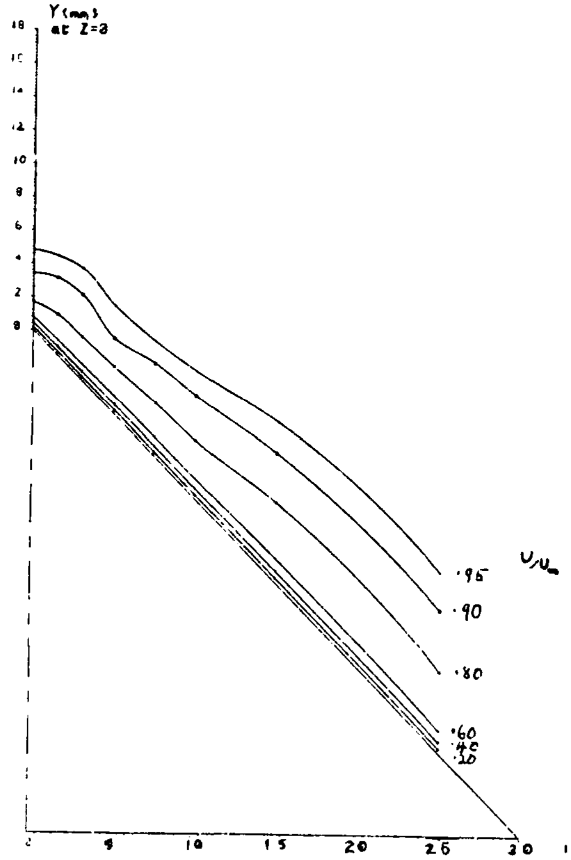
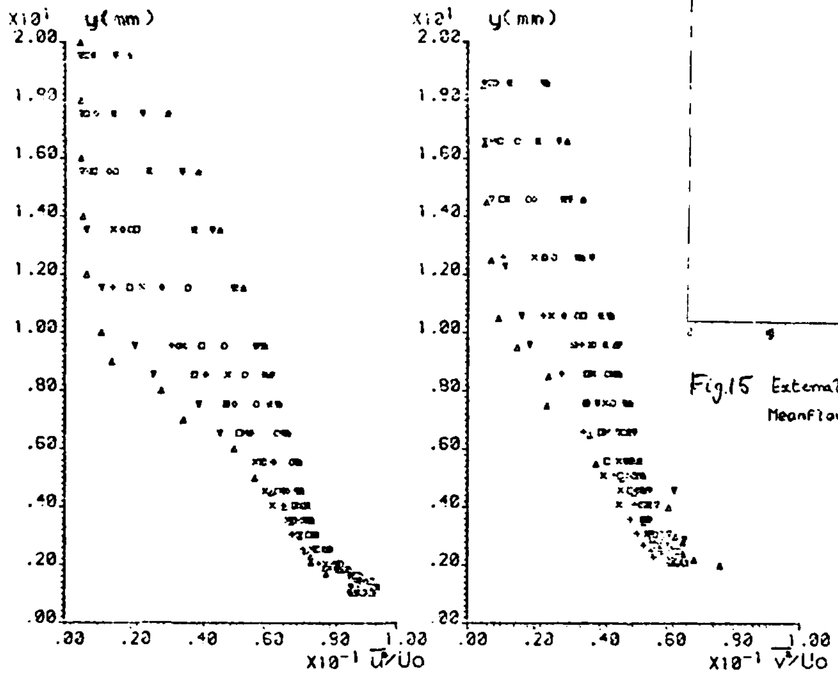


Fig.15 External Streamwise Corner Flow, Turbulent. Meanflow isovels at X= .479 m, and U_∞ =13.38m/s (Sharp L.E., fixed transition) (Patel)



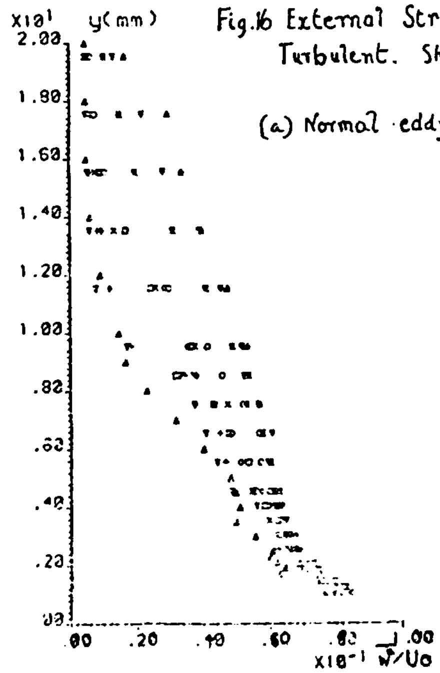
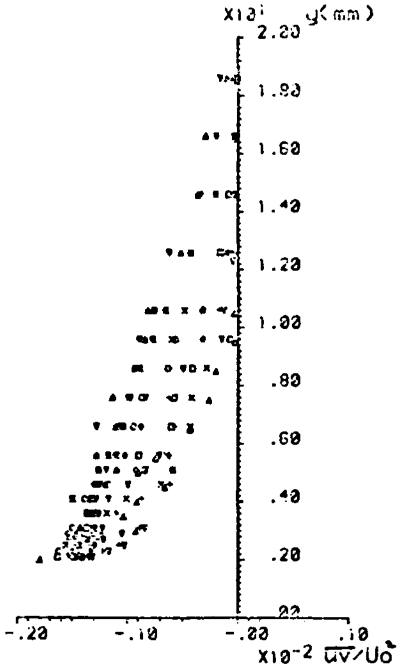
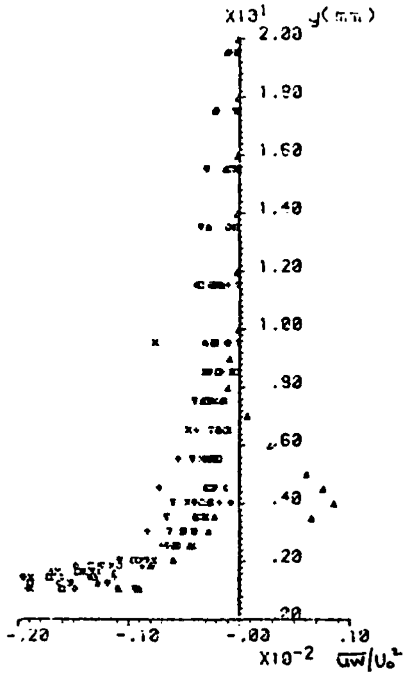


Fig.16 External Streamwise Corner Flow.
Turbulent. Sharp.L.E., fixed transition (Patel)

(a) Normal eddy stresses, X= .7328 m, U_∞ =13.30 m/s

	Z/mm.
△	.0
▽	1.5
+	3.0
×	5.0
□	7.5
◇	10.0
○	15.0
✕	25.0
▲	50.0
▼	75.0





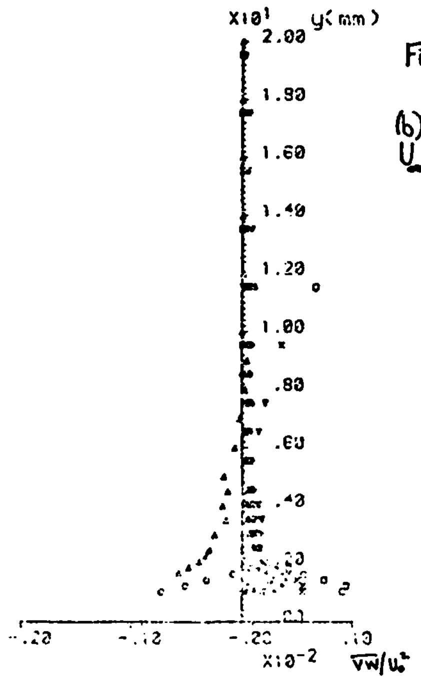


Fig.16 (cont.)

(b) Shear eddy stresses X=7328m
U_∞=13.30m/s

Z/mm.	
△	.0
▽	1.5
+	3.0
×	5.0
□	7.5
◇	10.0
○	15.0
✱	25.0
▲	50.0
♦	75.0

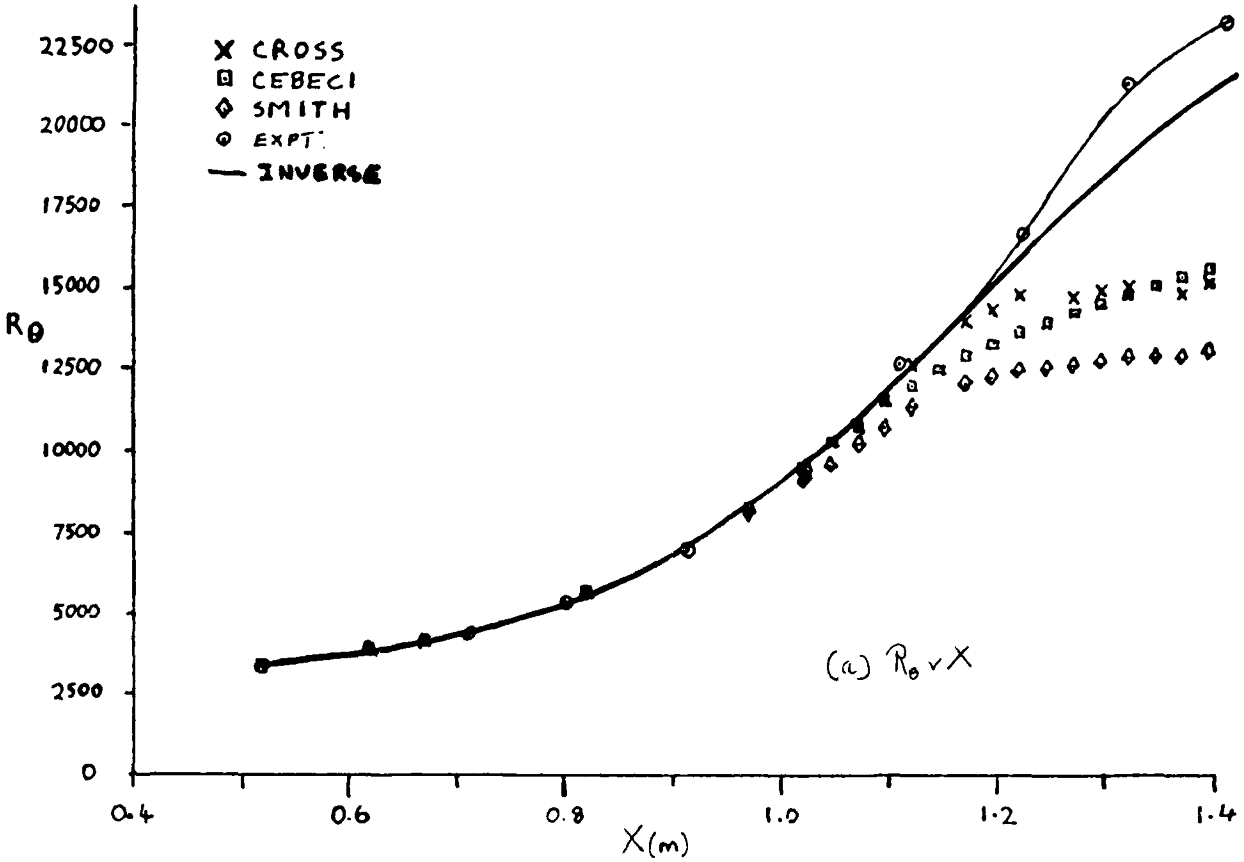


Fig. 17. Comparisons of Results of P.D. Smith's Inverse Method and of other Methods with NLR Experimental Results

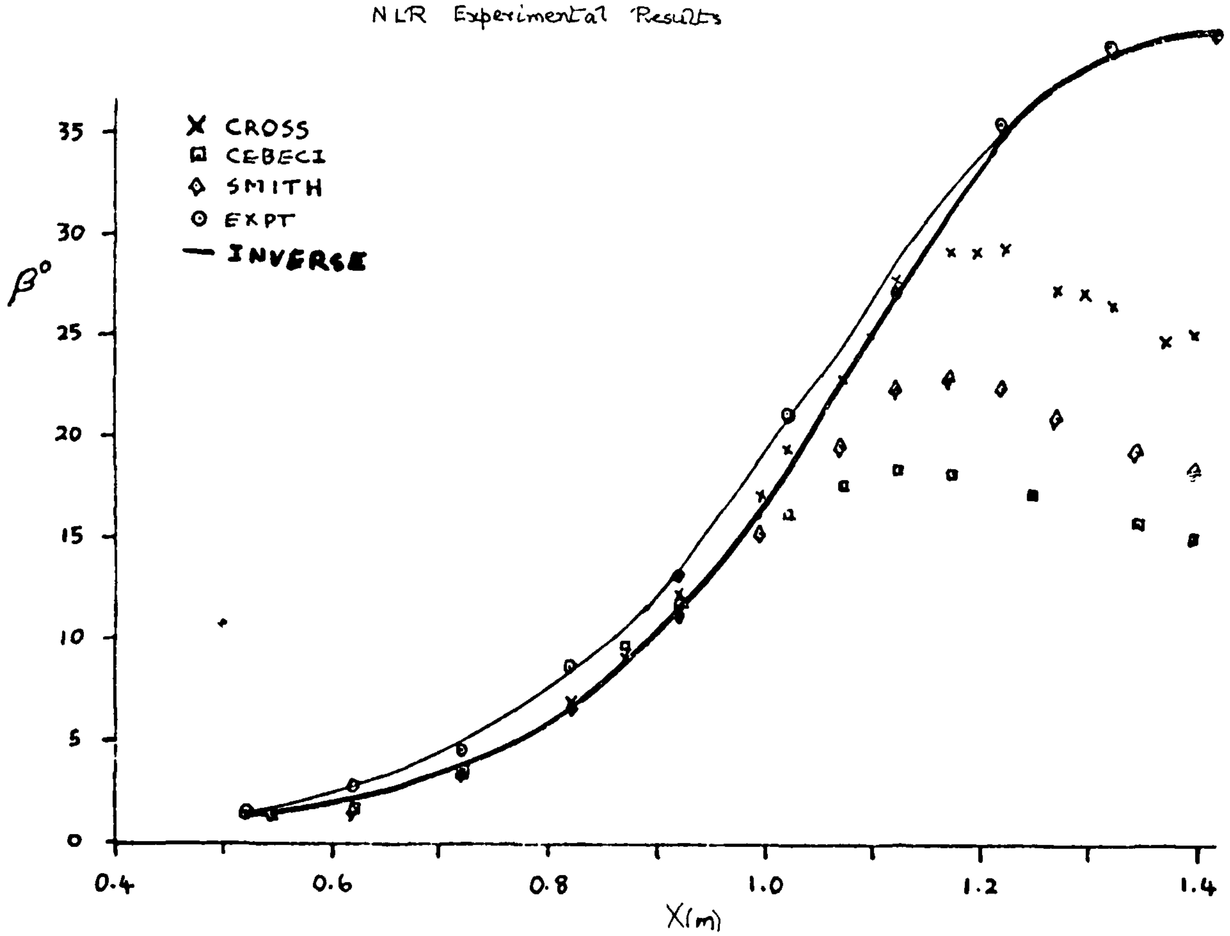


Fig. 17 (b) β° v X

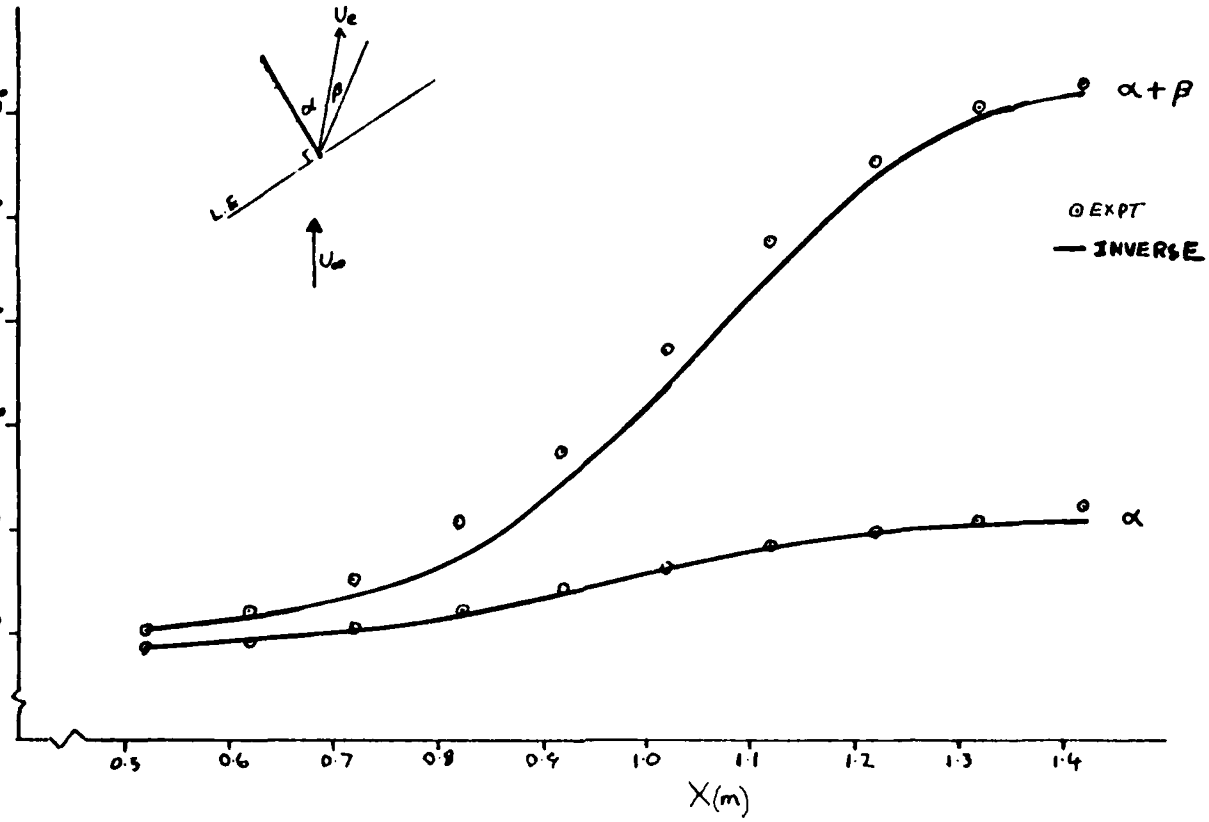


Fig. 17(c). α and $\alpha+\beta$ v X

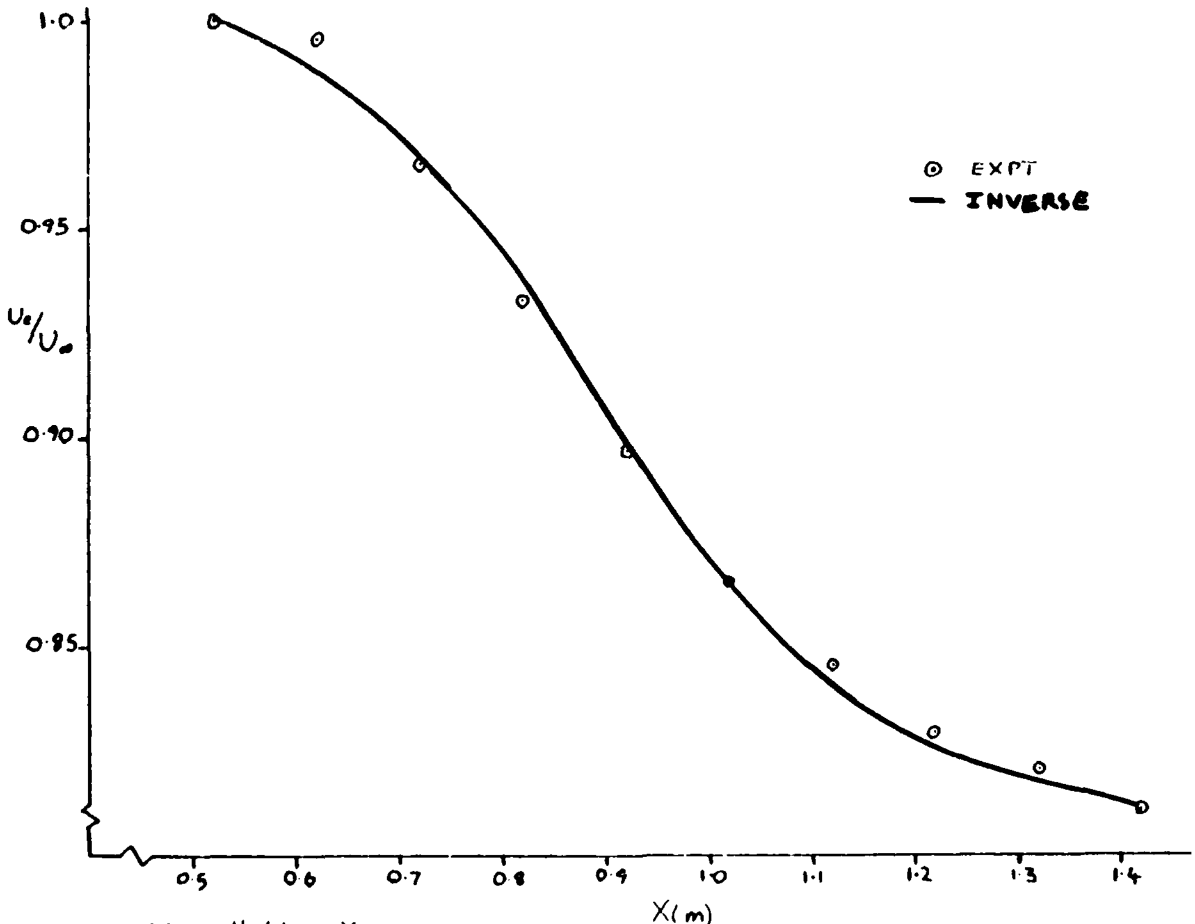


Fig 17(d) U_e/U_∞ v X

PROBLEMS AND OPPORTUNITIES WITH THREE-DIMENSIONAL BOUNDARY LAYERS

by

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1.0 INTRODUCTORY REMARKS

The purpose of this paper is to review research activity concerned with three-dimensional boundary layers and, as a result, to provide a view of present understanding and abilities. The subject is relevant to a wide range of engineering and environmental applications and the present discussion is presented with emphasis on the flows over the components of airplanes, missiles and ships. In each case, the objective is to develop understanding of the flow characteristics and of the influence of important parameters including geometry, Reynolds number and Mach number, and to make use of this understanding to improve design procedures. This last aspect frequently involves a calculation method which incorporates essential physics and represents the more important flow properties for a range of the parameters.

Physical understanding is usually obtained from experimental investigations and close interaction between experiments and the development of calculation methods is essential. The calculation method allows a useful framework with which to examine the experimental investigations and this approach is taken here. As a consequence, the following section considers basic equations and their solution and includes discussion of experimental information of direct relevance to the specification of Reynolds stresses and the related turbulent heat flux term. The calculation methods include the use of differential and integral equations and may be viewed as vehicles which permit the interpolation of experimental information and, with considerable care, their extrapolation. A similar approach was taken by Cebeci, Stewartson and Whitelaw (1984) in their consideration of the calculation of two-dimensional flows and it is useful to examine their conclusions since it is to be expected that our knowledge of two-dimensional flows will be better than that of three-dimensional flows.

The general conclusion of Cebeci et al. for two-dimensional flows was that calculation methods can and should be improved. This applied to the solution of inviscid, boundary-layer and Navier-Stokes equations and here we emphasize the second. The solution of inviscid-flow equations is not the subject of this review but the topic is considered in connection with the interaction of solutions of boundary-layer and inviscid-flow equations. Navier-Stokes equations are likely to be a major contributor to two-dimensional flows, although numerical uncertainties remain to be satisfactorily resolved; they are, as a consequence, less likely to be of assistance to three-dimensional flows although their use in conjunction with interactive procedures may prove to be of considerable value. With two-dimensional boundary-layer flows, integral procedures appeared to Cebeci et al. to be in favor but a tendency to differential methods was evident. As will be shown, integral methods are still used for three-dimensional boundary-layer flows, but are already less satisfactory than differential methods especially where negative cross-flows are involved. Emphasis on efficiency of calculation is essential with both approaches and is closely related to numerical accuracy.

Experimental emphasis on transition, separation, shock-wave/boundary-layer interactions and on the trailing-edge flows of single and multielement airfoils is required. The same problems exist in three-dimensional flows and the same emphases are needed. It should be recognized that measurements in three-dimensional flows are more difficult, if only because gradients exist in more than one plane, and are required to be more extensive in number in order to characterize the flow. Thus, for example, the number of significant components of the Reynolds stress tensor is increased in a three-dimensional flow and turbulence-model requirements are accordingly more complex. As will be shown, the number of detailed experimental investigations of wing flows is considerably less than for airfoil flows. Similarly, investigations of the flow over bodies of revolution at angle of attack are few even though the separation patterns are more complex and would suggest that research would be especially fruitful. The reasons for the smaller number of investigations of three-dimensional flows include the more demanding and more costly nature of the measurements. Thus, more rapid returns may appear to be available in two-dimensional flows, such as those recommended by Cebeci et al. An important purpose of this article is, therefore, to identify the more urgent three-dimensional flow problems and to urge that related experimental investigations be undertaken in the near future.

The sequence of discussion is similar to that of Cebeci et al. so that the conservation equations and their solution are considered in the next section which is followed by an appraisal of the present state of knowledge. The equations include differential and integral forms of the three-dimensional boundary-layer equations together with the reduced forms required to determine initial conditions. A subsection is devoted to coordinate systems which are required to facilitate accurate and efficient solutions for the flows around the various shapes under consideration and particularly to deal with the differences which exist, for example, between a wing and the bulbous bow of a modern ship. Assumptions are required to represent the turbulent flux terms in differential and in integral equations and are based entirely on experimentally determined information. The value of a model also involves numerical aspects and bases for choice are discussed in terms of both physical and numerical features in a corresponding subsection. Details of the various possible combinations of numerical assumptions and algorithms are not discussed, but the special problems associated with viscous-inviscid interactions and three-dimensional boundary-layer separation are considered.

The appraisal of the results of previous studies of three-dimensional boundary layers is presented in two parts. The first is concerned with the knowledge made available as a consequence of experimental investigations and this is interpreted in terms of the implications for the type of component, for example

a wing, under investigation and also for possible calculation methods. The second considers the current abilities of calculation methods and this is achieved partly in relation to measurements. The paper ends with conclusions and recommendations. As in the appraisal, this section first considers generally applicable future needs prior to consideration of the specific needs of wings, missiles and ships.

2.0 BOUNDARY LAYER EQUATIONS AND THEIR SOLUTIONS

2.1 Equations and Initial Conditions

The three-dimensional boundary-layer equations have been considered by, for example, Squire (1957), Nash and Patel (1972) and Bradshaw et al. (1981). It is evident that the use of an appropriate coordinate system is essential and nonorthogonal systems provide the necessary flexibility. The three-dimensional boundary-layer equations for steady compressible laminar and turbulent flows may be expressed in non-orthogonal form as:

$$\frac{\partial}{\partial x}(\rho u h_2 \sin\theta) + \frac{\partial}{\partial z}(\rho w h_1 \sin\theta) + \frac{\partial}{\partial y}(\overline{\rho v} h_1 h_2 \sin\theta) = 0 \tag{2.1}$$

$$\rho \frac{u}{h_1}\frac{\partial u}{\partial x} + \rho \frac{w}{h_2}\frac{\partial u}{\partial z} + \overline{\rho v}\frac{\partial u}{\partial y} - \rho \cot\theta\, K_1 u^2 + \rho \csc\theta\, K_2 w^2 + \rho K_{12} uw$$

$$= -\frac{\csc^2\theta}{h_1}\frac{\partial p}{\partial x} + \frac{\cot\theta \csc\theta}{h_2}\frac{\partial p}{\partial z} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y} - \rho\overline{u'v'}\right) \tag{2.2}$$

$$\rho \frac{u}{h_1}\frac{\partial w}{\partial x} + \rho \frac{w}{h_2}\frac{\partial w}{\partial z} + \overline{\rho v}\frac{\partial w}{\partial y} - \rho \cot\theta\, K_2 w^2 + \rho \csc\theta\, K_1 u^2 + \rho K_{21}^2 uw$$

$$= \frac{\cot\theta \csc\theta}{h_1}\frac{\partial p}{\partial x} - \frac{\csc^2\theta}{h_2}\frac{\partial p}{\partial z} + \frac{\partial}{\partial y}\left(\mu \frac{\partial w}{\partial y} - \rho\overline{w'v'}\right) \tag{2.3}$$

$$\rho \frac{u}{h_1}\frac{\partial H}{\partial x} + \rho \frac{w}{h_2}\frac{\partial H}{\partial z} + \overline{\rho v}\frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{Pr}\frac{\partial H}{\partial y} + \mu\left(1 - \frac{1}{Pr}\right)\frac{\partial}{\partial y}\left(\frac{u_t^2}{2}\right) + c_p\rho\overline{T'v'} + c_p\overline{\rho'T'v'}\right.$$

$$\left. - u(\rho\overline{u'v'} + \overline{\rho'u'v'}) - w(\rho\overline{w'v'} + \overline{\rho'w'v'})\right] \tag{2.4}$$

Here $\overline{\rho v} = \rho v + \overline{\rho' v'}$ and $h_1(x,z)$ and $h_2(x,z)$ are the metric coefficients and θ represents the angle between the coordinate lines; $\theta = \pi/2$ for an orthogonal system. The parameters K_1 and K_2 are known as the geodesic curvatures of the curves z = constant and x = constant, respectively, and are given by

$$K_1 = \frac{1}{h_1 h_2 \sin\theta}\left[\frac{\partial}{\partial x}(h_2\cos\theta) - \frac{\partial h_1}{\partial z}\right], \qquad K_2 = \frac{1}{h_1 h_2 \sin\theta}\left[\frac{\partial}{\partial z}(h_1\cos\theta) - \frac{\partial h_2}{\partial x}\right] \tag{2.5}$$

The parameters K_{12} and K_{21} are defined by

$$K_{12} = \frac{1}{\sin\theta}\left[-\left(K_1 + \frac{1}{h_1}\frac{\partial\theta}{\partial x}\right) + \cos\theta\left(K_2 + \frac{1}{h_2}\frac{\partial\theta}{\partial z}\right)\right] \tag{2.6a}$$

$$K_{21} = \frac{1}{\sin\theta}\left[-\left(K_2 + \frac{1}{h_2}\frac{\partial\theta}{\partial z}\right) + \cos\theta\left(K_1 + \frac{1}{h_1}\frac{\partial\theta}{\partial x}\right)\right] \tag{2.6b}$$

The magnitude of the velocity vector u_t in the boundary layer is given by

$$u_t = (u^2 + w^2 + 2uw\cos\theta)^{1/2} \tag{2.7}$$

At the edge of the boundary layer, Eqs. (2.2) and (2.3) become

$$\rho_e\left(\frac{u_e}{h_1}\frac{\partial u_e}{\partial x} + \frac{w_e}{h_2}\frac{\partial u_e}{\partial z} - \cot\theta\, K_1 u_e^2 + \csc\theta\, K_2 w_e^2 + K_{12} u_e w_e\right) = -\frac{\csc^2\theta}{h_1}\frac{\partial p}{\partial x} + \frac{\cot\theta\csc\theta}{h_2}\frac{\partial p}{\partial z} \tag{2.8}$$

$$\rho_e\left(\frac{u_e}{h_1}\frac{\partial w_e}{\partial x} + \frac{w_e}{h_2}\frac{\partial w_e}{\partial z} - \cot\theta\, K_2 w_e^2 + \csc\theta\, K_1 u_e^2 + K_{21} w_e u_e\right) = \frac{\cot\theta\csc\theta}{h_1}\frac{\partial p}{\partial x} - \frac{\csc^2\theta}{h_2}\frac{\partial p}{\partial z} \tag{2.9}$$

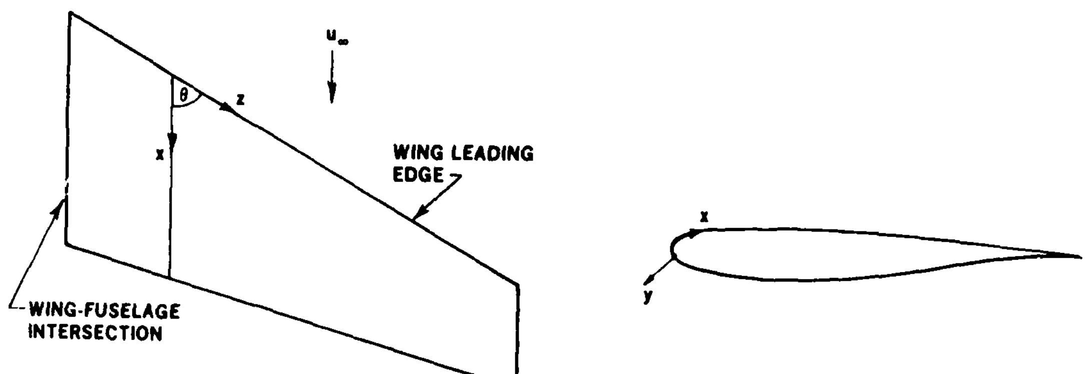
The boundary conditions appropriate to equations (2.1) to (2.7) may be expressed as

$$y = 0: \qquad u, v, w = 0 \qquad H = H_w(x,z) \quad \text{or} \quad \left(\frac{\partial H}{\partial y}\right)_w = -\frac{c_{p_w}}{k_w} q_w \tag{2.10a}$$

$$y = \delta: \qquad u = u_e(x,z), \qquad w = w_e(x,z) \qquad H = H_e \tag{2.10b}$$

Equations (2.1) to (2.4) are based on the assumption that the pressure is constant across the shear layer and stress gradients in directions parallel to the surface are negligible in comparison with those normal to the surface. These assumptions correspond to first-order boundary-layer theory or thin-shear-layer approximations and their validity decreases with increasing longitudinal curvature so that, with the relatively large curvature associated with many three-dimensional bodies, higher-order boundary-layer theory is required. Unfortunately, the main body of mathematical work on higher-order boundary-layer theory has been formulated in laminar flows because the relation between the velocity field and the stresses is known. For a discussion of "second-order" boundary-layer equations and thick boundary-layer equations the reader is referred to Patel (1982) and for the basis of the so-called fairly thin-shear-layer approximation in turbulent flow to Bradshaw (1975).

The solution of the above system of equations requires initial conditions in the (x,y)-plane at some $z = z_0$ and initial conditions in the (z,y)-plane at $x = x_0$. In some problems these conditions can be established with ease, and in others they require careful examination. In the case of a finite wing, see Fig. 1 for notation, the initial conditions usually correspond to those along the wing leading edge and to those along the wing-fuselage intersection. Near the leading edge of the wing, u and $\partial p/\partial x$ are equal to zero; this makes the x-momentum equation (2.2) singular along the stagnation line. However, differentiating Eq. (2.2) and taking advantage of approximate symmetry conditions,



(a) (b)

Figure 1. The notation for a swept wing: (a) wing-planform, (b) airfoil section.

$$\frac{\partial w}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial^2 u}{\partial x^2} = 0$$

and using Eqs. (2.9) and (2.10), we can write the appropriate boundary-layer equations along the stagnation-line of a wing as:

$$\rho h_2 \sin\theta u_x + \frac{\partial}{\partial z}(\rho w h_1 \sin\theta) + \frac{\partial}{\partial y}(\overline{\rho v} h_1 h_2 \sin\theta) = 0 \tag{2.11}$$

$$\rho \frac{u_x^2}{h_1} + \rho \frac{w}{h_2}\frac{\partial u_x}{\partial z} + \overline{\rho v}\frac{\partial u_x}{\partial y} + \rho w^2 (K_2 \csc\theta)_x + \rho K_{12} w u_x$$

$$= \rho_e \left[\frac{u_{xe}^2}{h_1} + \frac{w_e}{h_2}\frac{\partial u_{xe}}{\partial x} w_e^2 (K_2 \csc\theta)_x + K_{12} w_e u_{xe}\right] + \frac{\partial}{\partial y}\left[\mu \frac{\partial u_x}{\partial y} - \rho(\overline{u'v'})_x\right] \tag{2.12}$$

$$\rho \frac{w}{h_2}\frac{\partial w}{\partial z} + \overline{\rho v}\frac{\partial w}{\partial y} - \rho \cot\theta K_2 w^2 = \rho_e \left(\frac{w_e}{h_2}\frac{\partial w_e}{\partial z} - \cot\theta K_2 w_e^2\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial w}{\partial y} - \rho\overline{w'v'}\right) \tag{2.13}$$

$$\rho \frac{w}{h_2}\frac{\partial H}{\partial z} + \overline{\rho v}\frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{Pr}\frac{\partial H}{\partial y} + \mu\left(1 - \frac{1}{Pr}\right)\frac{\partial}{\partial y}\left(\frac{u_t}{2}\right)^2 + c_p(\rho\overline{T'v'} + \overline{\rho' T' v'}) - w(\rho\overline{w'v'} + \overline{\rho' w' v'})\right] \tag{2.14}$$

Here $u_x = \partial u/\partial x$, $u_{xe} = \partial u_e/\partial x$, $(K_2 \csc\theta)_x = \partial/\partial x(K_2 \csc\theta)$, $u_t = w$ and the equations are subject to the boundary conditions:

$$y = 0, \qquad u_x, v, w = 0, \qquad H = H_w(x,z) \quad \text{or} \quad \left(\frac{\partial H}{\partial y}\right)_w = -\frac{c_{p_w}}{k_w} q_w \tag{2.15a}$$

$$y = \delta, \qquad u_x = u_{xe}(x,z), \qquad w = w_e(x,z), \qquad H = H_e \tag{2.15b}$$

In general the boundary layer growing on the fuselage collides with the obstacle formed by the wing protruding from the fuselage and separates to form a horseshoe vortex starting from the leading edge and extending along the wing-fuselage intersection. The flow in this region is not of boundary-layer type but belongs to a class often referred to as boundary region (see Cebeci and Bradshaw, 1977). An account of the theoretical structure of boundary regions related to corner flows has been given by Rubin and Grossman (1971) and by Ghia (1975) while observations of the flow properties have been reported by Zamir and Young (1970) and El Gamal and Barclay (1978). A review of the available knowledge has been recently given by Zamir (1981). Although the general nature of the pressure-driven corner vortex is understood, its representation by equations and boundary conditions has not been demonstrated. It is evident that the flow in any cross-stream plane is characterized by two orthogonal gradients so that the boundary-layer assumptions are invalid. Calculations have been attempted by solving forms of the Navier-Stokes equations, for example by Baker and Orzechowski (1983) and Hah (1983), but little numerical testing of the solution procedures has been reported and their general validity is doubtful.

Until a more general and soundly based procedure for calculating the wing-root flow has been developed, it is necessary to determine corresponding initial conditions with the help of the infinite swept-wing assumptions, or the attachment-line equations, as employed by Cebeci, Kaups, Ramsey (1977) and by McLean (1977). In the former the spanwise flow is independent of the z-coordinate,

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial w}{\partial z} = \frac{\partial H}{\partial z} = 0 \, ,$$

and the continuity, x-momentum and energy equations reduce to the two-dimensional forms

$$\frac{\partial}{\partial x}(\rho u h_2 \sin\theta) + \frac{\partial}{\partial y}(\overline{\rho v} h_1 h_2 \sin\theta) = 0 \tag{2.16}$$

$$\rho \frac{u}{h_1}\frac{\partial u}{\partial x} + \overline{\rho v}\frac{\partial u}{\partial y} - \rho \cot\theta\, K_1 u^2 + \rho\csc_\theta\, K_2 w^2 + \rho K_{12} uw$$

$$= \rho_e \left(\frac{u_e}{h_1}\frac{\partial u_e}{\partial x} - \cot\theta\, K_1 u_e^2 + \csc\theta\, K_2 w_e^2 + K_{12} u_e w_e\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y} - \rho\overline{u'v'}\right) \tag{2.17}$$

$$\rho \frac{u}{h_1}\frac{\partial H}{\partial x} + \overline{\rho v}\frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{Pr}\frac{\partial H}{\partial y} + \mu\left(1 - \frac{1}{Pr}\right)\frac{\partial}{\partial y}\left(\frac{u_t^2}{2}\right) + c_p\overline{\rho T'v'} - u\overline{\rho u'v'} - w\overline{\rho w'v'}\right] \tag{2.18}$$

and the z-momentum equation becomes:

$$\rho \frac{u}{h_1}\frac{\partial w}{\partial x} + \overline{\rho v}\frac{\partial w}{\partial y} - \rho \cot\theta\, K_2 w^2 + \rho \csc\theta\, K_1 u^2 + \rho K_{21}^2 uw$$

$$= \rho_e \left(\frac{u_e}{h_1}\frac{\partial w_e}{\partial x} - \cot\theta\, K_1 w_e^2 + \csc\theta\, K_2 u_e^2 + K_{12} u_e w_e\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial w}{\partial y} - \rho\overline{w'v'}\right) \tag{2.19}$$

The boundary conditions for this case are the same as those given by Eq. (2.10) except that u_e and w_e are independent of z.

If we assume that w and $\partial p/\partial z$ are zero along the wing-fuselage intersection, we obtain equations similar to those along the stagnation line of the wing. On this occasion the spanwise equation is singular although differentiation with respect to z yields a nonsingular equation. After performing the necessary differentiation for Eq. (2.3) and taking advantage of the approximate symmetry conditions,

$$\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial^2 w}{\partial z^2} = 0 \, ,$$

and using Eqs. (2.8) and (2.9), the so-called attachment line equations in the chordwise direction can be written as

$$\frac{\partial}{\partial x}(\rho u h_2 \sin\theta) + \rho h_1 \sin\theta\, w_z + \frac{\partial}{\partial y}(\overline{\rho v} h_1 h_2 \sin\theta) = 0 \tag{2.20}$$

$$\rho \frac{u}{h_1}\frac{\partial u}{\partial x} + \overline{\rho v}\frac{\partial u}{\partial y} - \rho \cot\theta\, K_1 u^2 = \rho_e\left(\frac{u_e}{h_1}\frac{\partial u_e}{\partial x} - \cot\theta\, K_1 u_e^2\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y} - \rho\overline{u'v'}\right) \tag{2.21}$$

$$\rho \frac{u}{h_1}\frac{\partial w_z}{\partial x} + \frac{\rho}{h_2} w_z^2 + \rho u^2 (K_1 \csc\theta)_z + \overline{\rho v}\frac{\partial w_z}{\partial y} + \rho K_{21} u w_z$$

$$= \rho_e\left[\frac{u_e}{h_1}\frac{\partial w_{ze}}{\partial x} + \frac{w_{ze}^2}{h_2} + u_e^2 (K_1 \csc\theta)_z + K_{21} u_e w_{ze}\right] + \frac{\partial}{\partial y}\left[\mu\frac{\partial w_z}{\partial y} - \rho(\overline{w'v'})_z\right] \tag{2.22}$$

$$\rho \frac{u}{h_1}\frac{\partial H}{\partial x} + \overline{\rho v}\frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{Pr}\frac{\partial H}{\partial y} + \mu\left(1 - \frac{1}{Pr}\right)\frac{\partial}{\partial y}\left(\frac{u_t^2}{2}\right) + c_p(\overline{\rho T'v'} + \overline{\rho' T' v'}) - u(\overline{\rho u'v'} + \overline{\rho' u'v'})\right] \qquad (2.23)$$

Here $w_z = \partial w/\partial z$, $w_{ze} = \partial w_e/\partial z$, $(K_1 \csc\theta)_z = \partial/\partial z(K_1 \csc\theta)$, $u_t = u$ and the equations are subject to the boundary conditions

$$y = 0, \qquad u = v = 0, \qquad w_z = 0, \qquad H = H_w(x,z) \quad \text{or} \quad \left(\frac{\partial H}{\partial y}\right)_w = -\frac{c_{p_w}}{K_w} q_w \qquad (2.24a)$$

$$y = \delta, \qquad u = u_e(x,z), \qquad w_z = w_{ze}, \qquad H = H_e \qquad (2.24b)$$

In practice corner fillets may reduce the problem of separation and, in this case, the above approximations based on boundary-layer theory may represent the details of the flow a little better than the case in which flow has extensive and/or large separation.

For completeness, typical integral equations representing conservation of momentum are given below for a nonorthogonal system

$$\frac{1}{\rho_e u_{te}^2 h_1 h_2 \sin\theta}\left[\frac{\partial}{\partial x}(\rho_e u_{te}^2 h_2 \theta_{11} \sin\theta) + \frac{\partial}{\partial z}(\rho_e u_{te}^2 h_1 \theta_{12} \sin\theta)\right] + \frac{\delta_1^*}{h_1 u_{te}}\frac{\partial u_e}{\partial x} + \frac{\delta_2^*}{h_2 u_{te}}\frac{\partial u_e}{\partial z}$$
$$- K_1 \cot\theta\left(\frac{u_e}{u_{te}}\delta_1^* + \theta_{11}\right) + K_2 \csc\theta\left(\frac{w_e}{u_{te}}\delta_2^* + \theta_{22}\right)$$
$$+ K_{12}\left(\frac{u_e}{u_{te}}\delta_2^* + \theta_{12}\right) = \frac{c_{fx}}{2} \qquad (2.25)$$

$$\frac{1}{\rho_e u_{te}^2 h_1 h_2 \sin\theta}\left[\frac{\partial}{\partial x}(\rho_e u_{te}^2 h_2 \theta_{21} \sin\theta) + \frac{\partial}{\partial z}(\rho_e u_{te}^2 h_1 \theta_{22} \sin\theta)\right] + \frac{\delta_1^*}{h_1 u_{te}}\frac{\partial w_e}{\partial x} + \frac{\delta_2^*}{h_2 u_{te}}\frac{\partial w_e}{\partial z}$$
$$- K_2 \cot\theta\left(\frac{w_e}{u_{te}}\delta_2^* + \theta_{22}\right) + K_1 \csc\theta\left(\frac{u_e}{u_{te}}\delta_1^* + \theta_{11}\right)$$
$$+ K_{21}\left(\frac{w_e}{u_{te}}\delta_1^* + \theta_{21}\right) = \frac{c_{fz}}{2} \qquad (2.26)$$

where

$$\rho_e u_{te}\delta_1^* = \int_0^\infty (\rho_e u_e - \rho u)dy \qquad \rho_e u_{te}\delta_2^* = \int_0^\infty (\rho_e w_e - \rho w)dy$$

$$\rho_e u_{te}^2\theta_{11} = \int_0^\infty \rho u(u_e - u)dy \qquad \rho_e u_{te}^2\theta_{12} = \int_0^\infty \rho w(u_e - u)dy \qquad (2.27)$$

$$\rho_e u_{te}^2\theta_{21} = \int_0^\infty \rho u(w_e - w)dy \qquad \rho_e u_{te}^2\theta_{22} = \int_0^\infty \rho w(w_e - w)dy$$

$$c_{f_x} = \frac{2\tau_{w_x}}{\rho_e u_{te}^2} \qquad c_{f_z} = \frac{2\tau_{w_z}}{\rho_e u_{te}^2}$$

It is easy to see that, with two conservation equations, two unknown shear stresses and the unknown dependent variables of Eq. (2.27), additional information is required and usually takes the form of auxiliary equations. Swafford (1983) has provided derivations of the above equations for a time-dependent flow and a discussion of requirements of auxiliary equations.

2.2 Coordinate System

The choice of a coordinate system is particularly important in a three-dimensional flow and a system of inviscid streamlines and their orthogonal trajectories on the surface is often used. As shown in Fig. 2, the projection of the freestream velocity vector on the surface is aligned with the surface coordinate x, and the velocity component along the z-axis, referred to as the crossflow velocity is zero at the edge of the boundary layer. The x-momentum equation (2.2) (with $\theta = \pi/2$) is referred to as the streamwise momentum, and z-momentum equation (2.3) is referred to as the cross-flow momentum equation.

The streamline coordinate system is widely applicable but requires the solution of potential-flow equations for each configuration. An alternative is the geometry-oriented or body-fitted coordinate system and in many cases this is simpler and, therefore, to be preferred. It is not, however, generally applicable and can introduce substantial numerical problems. A body of revolution at incidence, for

example, has a stagnation point removed from the axis of symmetry, as shown on Figure 3, and the solution of the equations on the leeside involves a singularity on the axis of symmetry. Transformations, as described by Cebeci, Khattab and Stewartson (1980), can be used to overcome this problem as can simple but less accurate methods, such as that of Wang (1970).

A nonorthogonal coordinate system is particularly appropriate to the representation of geometries such as wings or ship hulls: an example is shown on Figure 4 where the nonorthogonal system has clearly led to a grid distribution which is likely to adequately represent the important flow characteristics. Figure 5, in contrast, shows an orthogonal coordinate system and a resulting grid distribution which is adequate in the vicinity of the wing root and leading edge but less adequate in the vicinity of the wing tip and trailing edge. In some cases, orthogonal systems can be used, as demonstrated by McLean (1977), who used a surface-fitted orthogonal system formed by spanwise lines connecting constant-percentage chordwise-surface distance from the attachment line of a wing to their orthogonal trajectories. This scheme still leads to potential difficulties in the regions of the leading edge and root and of the trailing edge and tip but the simpler calculations associated with the orthogonal system offer considerable compensation.

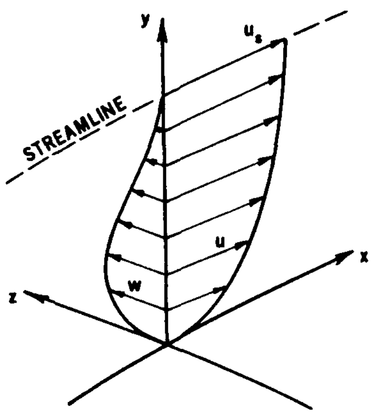


Figure 2. Streamline coordinate system.

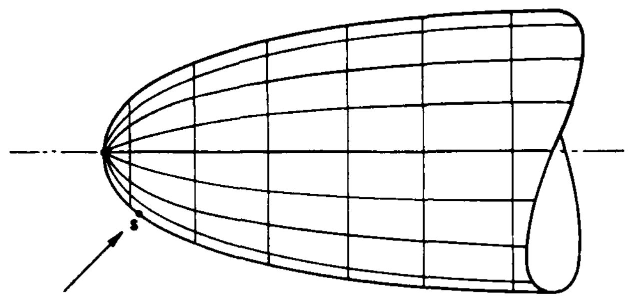


Figure 3. Body of revolution at incidence.

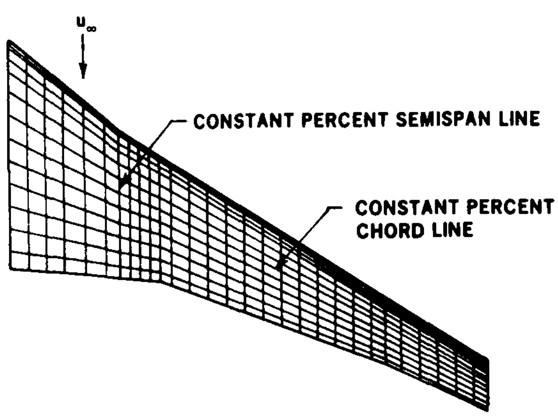


Figure 4. Nonorthogonal grid on a finite wing.

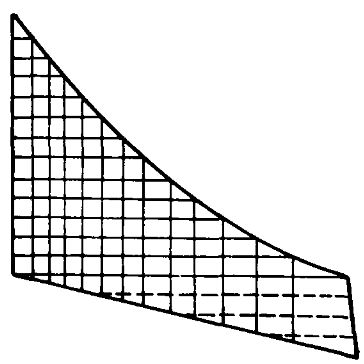


Figure 5. Orthogonal grid on a finite wing.

With either of the coordinate systems of Figures 4 and 5, it is desirable to express the boundary-layer equations in transformed variables to reduce the boundary-layer growth and the number of x- and z-stations on the body. A convenient transformation for this purpose is that used by Cebeci and his associates and similar to the Falkner-Skan transformation for two-dimensional flows. For three-dimensional flows, and in relation to Eqs. (2.1) to (2.4) or to the particular form given by Eqs. (2.11) to (2.14) and (2.16) to (2.23), the transformed coordinates, with $ds_1 = h_1 dx$, are given by

$$x = x, \qquad z = z, \qquad d\eta = \left(\frac{u_e}{\rho_e \mu_e s_1}\right)^{1/2} \rho dy \tag{2.28}$$

and a two-component velocity potential is introduced such that

$$\rho u h_2 \sin\theta = \frac{\partial\psi}{\partial y}, \qquad \rho w h_1 \sin\theta = \frac{\partial\phi}{\partial y}, \qquad \overline{\rho v} h_1 h_2 \sin\theta = -\left(\frac{\partial\psi}{\partial x} + \frac{\partial\phi}{\partial z}\right) \tag{2.29}$$

In addition, we define dimensionless ψ and θ by

$$\psi = (\rho_e \mu_e u_e s_1)^{1/2} h_2 \sin\theta f(x,z,\eta), \qquad \theta = (\rho_e \mu_e u_e s_1)^{1/2} \frac{u_{ref}}{u_e} h_1 \sin\theta g(x,z,\eta) \tag{2.30}$$

The transformed coordinates used for the stagnation-line equations are given by

$$x = x, \qquad z = z, \qquad d\eta = \left(\frac{u_{xe}}{\rho_e \mu_e h_1}\right)^{1/2} \rho dy , \tag{2.31}$$

the two-component vector potential by

$$\rho u_x h_2 \sin\theta = \frac{\partial\psi}{\partial y} , \qquad \rho w h_1 \sin\theta = \frac{\partial\phi}{\partial y} , \qquad \overline{\rho v} h_1 h_2 \sin\theta = -(\psi + \frac{\partial\phi}{\partial z}) , \tag{2.32}$$

and the dimensionless ψ and θ by

$$\psi = (\rho_e \mu_e u_{xe} h_1)^{1/2} h_2 \sin\theta f(z,\eta), \qquad \phi = (\rho_e \mu_e u_{xe} h_1)^{1/2} \frac{u_{ref}}{u_{xe}} h_1 \sin\theta g(z,\eta) \tag{2.33}$$

To transform chordwise attachment-line equations such as those given by Eqs. (2.20) to (2.23), we use the transformed coordinates given by Eq. (2.28) and define the two-component vector potential by

$$\rho u h_2 \sin\theta = \frac{\partial\psi}{\partial y} , \qquad w_z \rho h_1 \sin\theta = \frac{\partial\phi}{\partial y} , \qquad \overline{\rho v} h_1 h_2 \sin\theta = - (\frac{\partial\psi}{\partial x} + \phi) \tag{2.34}$$

with ψ and ϕ still given by Eq. (2.30).

2.3 Turbulence Models

The equations of Section 2.1 are Reynolds averaged as a consequence of an inability to solve the time-dependent, three-dimensional Navier-Stokes equations with sufficient resolution. In principle, it is possible to solve the time-dependent equations for the larger scales and to solve time-averaged equations for the smaller scales with suitable modelling of the small-scale structure. This possibility cannot be realized within the near future even for two-dimensional flows and there is unlikely to be any alternative to modelling the Reynolds-stress terms in the averaged equations. Of course, there are many alternative models which may be used and the appropriate choice depends on the flow under consideration and on the required accuracy and cost of the results.

Some criteria for the choice of a turbulence model are evident. It is, for example, evident that an algebraic specification for eddy viscosity will allow accurate prediction of mean-velocity components at low cost, provided the coefficients of the algebraic equation are obtained from flows similar to that under examination. In a similar way, and in connection with integral methods, correlations for drag coefficient and dissipation integral can allow accurate results to be obtained for flows which mean that the equations are being asked to interpolate known results. This is demonstrated very clearly by the calculations reported at the 1968 Stanford Conference (Coles and Hirst, 1969). The correct representation of flows where the turbulent structure is far from equilibrium, for example where boundary layers in strong adverse pressure gradients or asymmetric flows where the locations of zero shear stress and zero velocity gradient are not coincident, requires, however, that the transport of the Reynolds stresses be considered. This necessarily involves a large number of assumptions and inability to quantitatively evaluate each assumption, with larger uncertainty and greater computational cost. In principle, and usually in practice, it also implies applicability to a wider range of flows.

More specific criteria, usually for two-dimensional flows, can be deduced from the extensive literature which includes the reviews of Marvin (1983), Kline et al. (1981) and Rodi (1980). Of more immediate relevance, Cebeci and Meier (1979) examined a range of two-dimensional boundary-layer flows and suggested that the uncertainties introduced by boundary conditions may be greater than the differences due to the change from an eddy-viscosity assumption based on an algebraic equation to one based on transport equations for turbulence kinetic energy and dissipation rate. This suggestion is also likely to be appropriate for models based on transport equations for the Reynolds stresses, particularly since the dissipation equation is common to all methods involving the solution of transport equations, but should not be extrapolated from the range of boundary-layer flows considered by Cebeci and Meier. It is also dependent upon the manner in which the near-wall region is linked to the wall and to the nature of the near-wake flow. It is likely, for example, that eddy-viscosity models are increasingly inappropriate as separation is approached; on the other hand, and as demonstrated by Nakayama (1984) and Adair, Thompson and Whitelaw (1984), the normal pressure gradient may control the flow to a greater extent than the turbulent structure and emphasis would be better directed to the inclusion of the normal pressure gradient in the conservation equations and in the near-wall assumptions.

In a sequel to the paper of Cebeci and Meier, Cebeci and Chang (1982) reported calculations for the infinite swept wing of van den Berg and Elsenaar (1972) and for the three-dimensional flow around an airfoil located normal to a plate investigated by East and Hoxey (1969). Again, results were obtained with two eddy-viscosity models based on an algebraic equation and on transport equations, and again it was concluded that the turbulence model was unimportant. Apart from the near-separation results for the flow of East and Hoxey, both models led to mean-velocity and skin-friction data in close agreement with the measurements: with this proviso, the law of the wall served as a satisfactory wall-boundary condition. A sample of the results obtained with the algebraic eddy-viscosity formulation for these two flows is given in Figure 6.

Again, it could be pointed out that stress transport equations could lead to improved results and especially to the ability to better represent flows with strong pressure gradients. In addition, transition remains a problem in three-dimensional flows and a well-founded stress-transport model could permit extension to include low Reynolds-number terms. A glance at the unmodelled form of the Reynolds-stress equation, written here in tensor notation and in terms of mass-weighted averages,

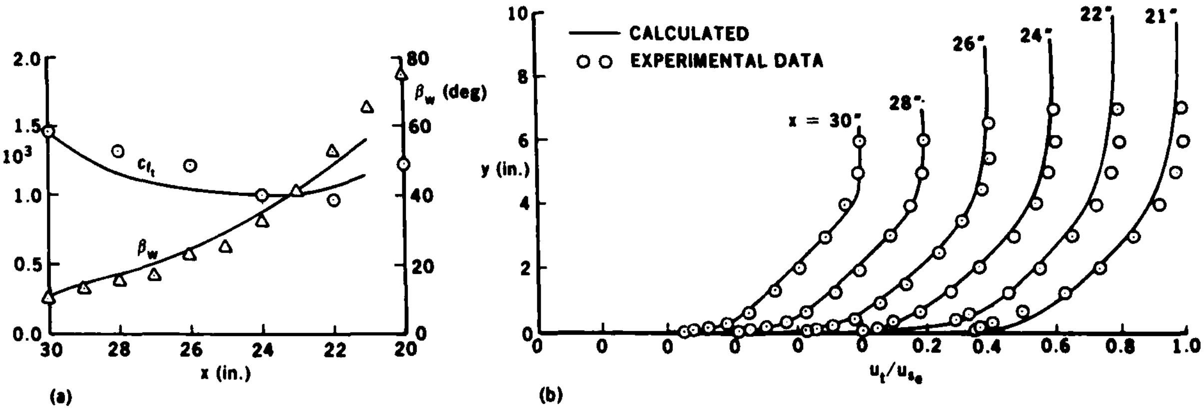


Figure 6. Results with the model of Cebeci and Smith. (a) Infinite swept wing of van den Berg and Elsenaar. (b) East and Hoxey flow.

$$\bar{\rho}\tilde{u}_k \frac{\partial}{\partial x_k} \widetilde{u_i''u_j''} = - \frac{\partial}{\partial x_k} \{\bar{\rho}\, \widetilde{u_i''u_j''u_k''}\} - \overline{u_i''}\frac{\partial \bar{p}}{\partial x_j} - \overline{u_j''}\frac{\partial \bar{p}}{\partial x_i} + \{\overline{u_j''\frac{\partial p'}{\partial x_i}} + \overline{u_i''\frac{\partial p'}{\partial x_j}}\} - \bar{\rho}\, \widetilde{u_i''u_k''}\frac{\partial \tilde{u}_j}{\partial x_k}$$

$$- \bar{\rho}\, \widetilde{u_j''u_k''}\frac{\partial \tilde{u}_i}{\partial x_k} - \{\overline{\tau_{\ell i}'\frac{\partial u_j''}{\partial x_\ell}} + \overline{\tau_{ij}''\frac{\partial u_i'}{\partial x_\ell}}\} \qquad (2.35)$$

demonstrates the extent of modelling required and the extent to which assumptions can be directly evaluated. A similar equation for the heat-flux balance and an even more intractable equation for the turbulent dissipation rate makes it very clear that a point of diminishing return has been reached and passed as far as three-dimensional boundary layers is concerned. For the calculation of fully turbulent flows it is likely that algebraic eddy-viscosity formulations, such as that of Cebeci and Smith (1974) and extended to three-dimensional flows by Cebeci (1978), are to be preferred. As an example, this algebraic eddy-viscosity formulation for a nonorthogonal coordinate system is provided below:

In the so-called inner region of the boundary layer ε_m is defined by the following formula

$$(\varepsilon_m)_i = L^2[(\frac{\partial u}{\partial y})^2 + (\frac{\partial w}{\partial y})^2 + 2\cos\theta\,(\frac{\partial u}{\partial y})(\frac{\partial w}{\partial y})]^{1/2}$$

where

$$L = 0.4y[1 - \exp(-y/A)], \qquad A = 26\,\frac{\nu}{u_\tau}\,(\frac{\rho}{\rho_w})^{1/2}, \qquad u_\tau = (\frac{\tau_{tw}}{\rho_w})^{1/2}$$

$$\tau_{tw} = \mu_w\,[(\frac{\partial u}{\partial y})_w^2 + (\frac{\partial w}{\partial y})_w^2 + 2\cos\theta(\frac{\partial u}{\partial y})_w(\frac{\partial w}{\partial y})_w]^{1/2}$$

In the outer region ε_m is defined by the following formula

$$(\varepsilon_m)_o = 0.0168\left|\int_0^\infty (u_{te} - u_t)dy\right|$$

It is of interest to note that the experimental investigations of van den Berg and Elsenaar (1972) and East and Hoxey (1969) were limited to mean-flow properties. As will be shown in Section 3, swept-wing investigations have recently been reported with low and transonic freestream velocities and have made use of hot-wire and laser velocimetry so that some turbulence information is available. Similarly, recent investigations of the flow around missile-like geometries have included turbulence information. Undoubtedly, the turbulence-model question should be reappraised in relation to these data but it seems unlikely that the conclusion of the previous paragraph will be altered, especially since the deviations from equilibrium which suggest the use of stress-transport equations are usually found as separation is approached and here it seems more important to correctly represent the effect of the normal pressure gradients. It is clear from the results of Nakayama (1984) and Adair et al. (1984) that, once the normal pressure gradients have been correctly represented, the deficiencies of eddy-viscosity models will become evident and should be addressed.

The numerical methods for solving transport equations for turbulence properties are also relevant to the present discussion. Algebraic eddy-viscosity equations do not require the solution of additional differential equations and, with simple equations of the type used by Cebeci and Smith, do not stiffen the equations and the solution of turbulent-flow equations involves little more effort than the solution of laminar-flow equations. With the common two-equation turbulence models, additional computational effort is required. With the Box scheme of Keller (1970) and two-dimensional flows, for example, the two differential equations imply that the number of equations is increased from three first-order equations

to seven so that the time for a converged solution increases by a factor of three. The Reynolds stress transport equations imply four additional differential equations, which lead to eleven first-order equations, increasing the computer time tenfold compared to the algebraic eddy-viscosity model. The larger times associated with the transport equations are moderately important but, when the solutions of the viscous-flow equations is interacted with solutions of the inviscid-flow equations, they can become of considerable importance, especially when, as frequently happens in practice, results are required for a number of configurations and angles of attack.

The problem of transition has been left to last because of the difficulties associated with its solution. It is important to wind or water tunnel testing which is normally conducted at Reynolds numbers which are less than those of the airplane component or ship. It may also control the extent or existence of separation bubbles. The following paragraphs discuss the subject in two parts. First, we will describe the methods used for the prediction of transition and comment on their merits. Secondly, the expressions used within the framework of turbulence models are considered. The available information on the implications of transition for separating and reattaching flows are also briefly considered by reference to the work of Cebeci and Schimke (1983).

As in two-dimensional flows, the transition in three-dimensional flows can also be calculated by using empirical methods. Most of these correlations come from the studies performed on swept laminar-flow-control (LFC) wings. To calculate the transition near the leading edge, it is necessary to analyze the flow in the streamwise as well as in the cross-flow directions. Growth of disturbances in the streamwise direction is assumed to occur when the value of R_{θ_s} exceeds that given by the formula

$$R_{\theta_s} = [A + B \frac{\theta_s^2}{u_{te}} (\frac{\partial^2 u_s}{\partial y^2})_w]^3$$

Here θ_s denotes the streamwise momentum thickness, u_s, the streamwise velocity component. The momentum thickness Reynolds number is defined by $u_{te}\theta_s/\nu$. The parameters A and B are empirical constants which need to be determined.

For cross-flow or secondary-flow stability, growth of disturbances is said to occur if R_n exceeds that given by the formula,

$$R_n = C + D \frac{y_{0.1}^2}{(u_n)_{max}} (\frac{\partial^2 u_n}{\partial y^2})_w ,$$

Here u_n is the cross-flow velocity, $y_{0.1}$ is the boundary-layer thickness determined by the location where u_n is equal to one tenth of its maximum value $(u_n)_{max}$. The Reynolds number R_n is defined by $(u_n)_{max} y_{0.1}/\nu$. Again the parameters C and D denote empirical constants.

Transition in two-dimensional flows can also be calculated from a combination of linear stability theory and the so-called e^n-method. For given velocity profiles, the amplification rates are computed and summed along the body until they reach approximately 20,000 ($\approx e^9$) where transition is said to occur. This method was originally suggested by van Ingen (1956) and A.M.O. Smith (1956) for two-dimensional flows and has been reasonably satisfactory. There have been no other "theoretical" methods. Its extension to three-dimensional flows has not received wide attention as it did in two-dimensional flows for two main reasons. First, the solution of the three-dimensional boundary-layer equations is more difficult and with incidence the flowfield becomes complex due to reversals in the spanwise or circumferential velocity. Secondly, the stability calculations are extremely difficult; the additional requirement that α and β must be obtained so that $\partial\alpha/\partial\beta$ is real, as discussed by Cebeci and Stewartson (1980) and Nayfeh (1980), introduces difficulties into the eigenvalue problem. Studies utilizing these ideas were first applied to a rotating disk by Cebeci and Stewartson by using spatial amplification theory, and the n-factor was computed to be around 20, which is quite high compared to the accepted value of 9 or 10. The reason for this discrepancy was attributed to the Orr-Sommerfeld equation which when written for the Cartesian coordinates, ignored the flow divergence and the Coriolis force. The effect of neglected terms was studied by Malik, Wilkinson and Orszag (1981) who found that the Coriolis forces have much smaller effect than the flow divergence. Including both effects, the calculated critical Reynolds number was found to be 287, a value very close to the latest measurements of Wilkinson and Malik (1983) and the n-value was 10.7, a value in line with results obtained on different configurations.

Similar ideas have also been applied to NASA-sponsored research related to LFC on wings by Strokowski and Orszag (1977), Malik (1982), and by Mack (1979). All these studies used temporal amplification theory and have led to useful methods for estimating the minimum suction rates necessary to maintain laminar flow on the wing, as well as predicting transition in compressible flows.

The three-dimensional stability calculations using spatial amplification theory and e^n-method has been investigated on bodies of revolution at incidence by Cebeci and Stewartson (1983a). The results have been compared with the experimental data of Meier and Kreplin (1980) obtained on a prolate spheroid at $\alpha = 10°$. As shown in Figure 7, the calculated and experimental results are in good agreement, but severe computational problems were encountered and are presently under investigation.

Expressions have been recommended by Dhawan and Narasimha (1958) and Chen and Thyson (1971) to allow calculations to proceed through a two-dimensional transition region. They stem from Emmon's spot theory (1951) and essentially correlate the length of transitional region to Reynolds number and have been used in algebraic eddy-viscosity formulas such as that of Cebeci and Smith (1974). A related expression has been recommended by McDonald and Fish (1973) for similar purposes. As yet, recommendations have not been made for three-dimensional flows and more careful examinations of two-dimensional recommendations is desirable before further empirical functions are added to the existing expressions. For example, Narasimha (1983), Narasimha et al. (1982, 1984) have recently pointed out that the Chen-Thyson model is

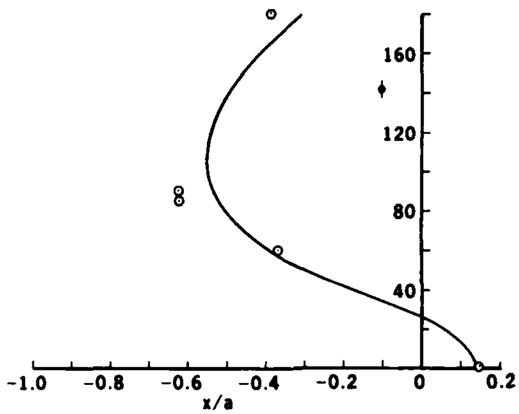


Figure 7. Comparison of results for a prolate spheroid at α = 10°. Solid line denotes the experimental transition line and the symbols the theoretical estimates of Cebeci and Stewartson.

not in accord with the observed intermittency distributions. The experiments of Kreplin, Vollmers and Meier (1982) provide a basis for the development of three-dimensional models.

As mentioned above, Cebeci and Schimke (1983) found that the location of transition plays an important role in the uniqueness of solutions involving separation bubbles on airfoils. It may be expected that similar phenomena will occur in three-dimensional flows and related investigations are clearly necessary.

2.4 Solution Procedures

It is hardly surprising that the solution of the three-dimensional boundary-layer equations is more difficult than of their two-dimensional counterparts. This is more so with differential equations and especially where the spanwise or circumferential velocity component may reverse. In such cases, numerical instabilities can result from integration in the direction opposite to the velocity and errors may occur unless special procedures are adopted, as described for example by Wang (1975), McLean (1977) and Bradshaw et al. (1981). Some aspects of these procedures are described in the following paragraphs and the reader is directed to the references cited for further explanations. Much of the present section is concerned with the interaction of solutions of the inviscid and viscous equations and the problem of three-dimensional separation. Integral methods can be used in interactive methods, as described for example by Cousteix et al. (1980), Gleyzes et al. (1984), Le Balleur (1984) and Whitfield (1978), and have so far only been extended to three dimensions for prescribed external velocity distribution. The present discussions are limited to differential equations since it is likely that they will be required to represent important flow phenomena such as negative crossflow velocities, the wall jets emerging from slotted flaps and essential details of separation.

McLean (1977) made use of an implicit finite-difference formulation to solve the three-dimensional boundary-layer equations and expressed spanwise derivatives in terms of second-order upwind differences when the crossflow velocities were positive and by a first-order scheme when they were negative. The numerical accuracy decreased as the magnitude and extent of negative crossflow velocities increased, but the choice of coordinate system (see Section 2.2) ameliorated the situation by reducing the magnitude and extent of the effective negative crossflow velocities. The same problem was addressed by Wang (1974a) in connection with bodies of revolution at angle of attack. He attempted to overcome it by variations in the computational mesh, but the results did not agree with the later calculations of Cebeci et al. (1981) and this demonstrates that the solution lies in the choice of an appropriate numerical method rather than in the computational details of implementation.

A variation of the ADI scheme of Nash and Scruggs (1976) has been used by Patel and his coworkers and has proved to be popular. At first, the method was unsatisfactory in any region of rapidly growing boundary-layer thickness in the circumferential direction since corresponding derivatives were evaluated from first-order backward differences. In the modified method of Patel and Baek (1982) higher-order differencing has been introduced and the problem resolved.

Perhaps the most satisfactory numerical procedure for dealing with reversals in the crossflow velocity is the characteristic box approach of Cebeci and Stewartson (1978), which is based on the solution of the equations along local streamlines. It has been applied to a range of problems related to wings, bodies of revolution and ship hulls and these have been summarized by Bradshaw et al. (1981). A similar procedure has been used by Ragab (1982) for bodies of revolution at incidence.

2.5 Interaction Procedures

In many real problems, the freestream velocity distribution is unknown and must be determined from solutions of the potential-flow equations, first for the body shape and subsequently for this shape modified by displacement-thickness distributions obtained from solutions of boundary-layer equations. As an alternative to the addition of displacement thickness, an equivalent blowing velocity distribution may be used. The requirements of interactive procedures are now discussed in relation to two-dimensional flows, and can be generalized to three dimensions, the procedures in this case are considerably more difficult.

With a symmetrical airfoil at zero angle of attack the stagnation point occurs on the axis but if the airfoil is asymmetric or at finite angle of attack, the position of stagnation is determined by continuity of pressure at the trailing edge, i.e. the Kutta condition. An apparent solution to this problem is to determine the stagnation location from the potential-flow equations and to iterate with solutions of the boundary-layer equations. This process involves two derivatives, without compensation for integration, which occur in the calculation of the normal velocity at the surface and obtained by differentiating the displacement thickness and in the calculation of pressure gradient from the modified inviscid-flow solution. Furthermore, accurate integration of the boundary-layer equations must terminate with a singularity, if separation occurs, and further progress is only possible with the help of some imprecise approximation procedure. For these reasons it is desirable and sometimes necessary to adopt an inverse procedure whereby the displacement thickness is prescribed in the boundary-layer formulation: as a result the difficulty at separation is removed and the iterative procedure is more stable as it involves two integrations without any compensatory differentiation.

This inverse boundary-layer approach has been used by a number of investigators, such as Klineberg and Steger (1974), Horton (1974), Carter (1974, 1975), Williams (1975), Pletcher (1978), Cebeci (1976) and Cebeci, Keller and Williams (1979). These studies were made by prescribing displacement thickness or wall shear distributions and calculations past the separation point were achieved by using the approximation first suggested by Reyhner and Flügge-Lotz (1968). This approximation which was necessary to overcome the stability problem associated with negative u-velocity and is referred to as FLARE (Flügge-Lotz and Reyhner) by Williams (1975) neglects the $u(\partial u/\partial x)$ term in the region of negative u-velocity. As long as the recirculation region is small, there is no need to improve the solutions obtained with this approximation. However, as the size of the recirculation region increases, the application of the FLARE approximation becomes increasingly inaccurate, indicating the need for a procedure in which the longitudinal convection term is accounted for. There are several procedures proposed for this purpose, Carter (1975), Williams (1975) and Cebeci (1983). One successful scheme, called the DUIT (Downstream, Upstream Iteration) procedure, is due to Williams (1975) and requires several numerical sweeps through the recirculation region to reintroduce the longitudinal convective term. Thus, FLARE is used to compute an approximate solution in the recirculating region and, in successive sweeps of this region, the $u(\partial u/\partial x)$ term is progressively introduced until it is fully represented. Of course, for large regions of separation, the boundary-layer equations may be inappropriate and the Navier-Stokes equations should be solved; the size of the recirculation at which it is desirable to switch from boundary-layer to Navier-Stokes equations remains to be determined. A useful review of this subject has been provided by McDonald and Briley (1984).

An alternative solution procedure which allows the inclusion of the longitudinal convection term in the boundary-layer equations is to use the solutions of the two-dimensional unsteady boundary-layer equations which are often obtained by solving the governing equations in the x-direction at a given time. Such spatial sweeps may be thought of as successive iterations in a procedure for solving the steady problem corresponding, for example, to the successive sweeps of Williams' DUIT procedure. For this reason, unsteady boundary-layer calculations provide an alternative, and possibly simpler, means of computing steady separating and reattaching flows. The feasibility of this approach was explored recently by solving the two-dimensional time-dependent boundary-layer equations for a specified displacement thickness distribution, Cebeci (1983). The study used Keller's box method with the Mechul function formulation developed by Cebeci (1976).

So far, two separate special coupling procedures between the viscous and inviscid flow equations have been developed for two-dimensional flows, the first is due to LeBalleur (1977) and Carter (1979) and the second to Veldman (1981). In the first the solution of the boundary-layer equations is obtained by the standard method and a mass flux distribution $q^\circ(x) = u_e\rho\delta^*$ determined. If this initial calculation encounters separation, $q^\circ(x)$ is extrapolated to the trailing edge, the corresponding displacement thickness, $\delta^{*o}(x)$ is calculated, and one complete cycle of the viscous and inviscid calculation is performed. In general, this leads to two external velocity distributions, $u_{e_v}(x)$ derived from the inverse boundary-layer solution and $u_{e_i}(x)$ derived from the updated approximation to the inviscid velocity past the airfoil with the added displacement thickness. A relaxation formula is introduced to define an updated mass flux distribution,

$$q(x) = q^\circ(x)\{1 + \omega\,[\frac{u_{e_v}(x)}{u_{e_i}(x)} - 1]\} \tag{2.36}$$

where ω is a relaxation parameter and the procedure is repeated with this updated mass flux. The papers by LeBalleur (1984), Kwon and Pletcher (1984), Carter (1981), Carter and Vatsa (1982) are based on variations of this method.

In the second approach, the external velocity $u_e(x)$ and the displacement thickness $\delta^*(x)$ are treated as unknown quantities and the equations are solved simultaneously in the inverse mode and with successive sweeps over the airfoil surface. For each sweep, the external boundary condition for the boundary-layer equation is written as

$$u_e(x) = u_e^0(x) + \delta u_e(x) \tag{2.37}$$

where $u_e^0(x)$ is the inviscid velocity over the airfoil and δu_e is the perturbation due to the displacement thickness. In terms of thin airfoil theory, this perturbation velocity is written as

$$\delta u_e = \frac{1}{\pi} \int_{x_a}^{x_b} \frac{d}{d\xi}(u_e\delta^*)\,\frac{d\xi}{x - \xi} \tag{2.38}$$

where the interaction is confined between x_a and x_b. A discrete approximation for the Hilbert integral involved in this equation gives

$$\delta u_e = \sum_{j=1}^{n} c_{ij}(u_e\delta^*)_j \tag{2.39}$$

where $[c_{ij}]$ is a matrix of interaction coefficients defining the relationship between the displacement thickness and the external velocity. In the approach used by Cebeci and Clark (1984), Eqs. (2.37) and (2.39) are expressed in the form

$$u_{e_i} = u_{e_{oi}}(x) + \sum_{j=1}^{n} c_{ij}\left[(u_e\delta^*)_j - (u_{e_o}\delta^*)_j\right] \tag{2.40}$$

where $u_{e_{oi}}$ is the external velocity corresponding to a given displacement thickness distribution δ^*_{oi}. For a given location on the airfoil both u_{e_i} and δ^*_i are assumed to be unknown quantities and are computed by the inverse boundary-layer procedure of Cebeci (1976). Equation (2.40) provides the outer boundary condition for this solution, which can be written in the form

$$u_{e_i} - c_{ii}u_{e_i}\delta^*_i = g_i \tag{2.41}$$

where both c_{ii} and g_i are known quantities. In fact, c_{ii} depends on the airfoil geometry only, while g_i depends on the geometry and on the latest available approximations to the displacement thickness δ^*_j at each point on the airfoil. In this way, Eq. (2.41) provides the coupling between the viscous and inviscid calculation procedures.

Solutions to the three-dimensional boundary-layer equations have been obtained by Formery and Delery (1981) and Radwan and Lekoudis (1983), the latter application related to an infinite swept wing. In both cases distributions of displacement thicknesses were specified and an inverse procedure with FLARE approximation employed. It was shown that, as expected, the singularity at separation is removed by the use of the inverse procedure. The remaining step to a full interactive procedure is large and of the two approaches discussed above, that due to Veldman offers the only real possibility for extension to three dimensions.

2.6 Nature of Three-Dimensonal Separation

In two-dimensional steady flows, flow separation corresponds to the location where the wall shear vanishes. Two views of three-dimensional separation have been advanced; one, due to Lighthill (1963), states that the separation line is a limiting streamline passing through isolated singular points of the solution and the other, due to Maskell (1955), claims that the separation line is formed by the envelope of the limiting streamlines. The significance of the distinction is that calculations near the separation line should be regular in the former case, and calculation through separation would be possible, if additional information is available. The latter definition implies that calculated results near the separation line would be singular and, in particular, the skin-friction component normal to the envelope would become zero and the displacement thickness would grow to infinity. Experiments do not solve this dilemma since real flows tend to eliminate singularities. Flow visualization results do, however, suggest that separation lines are envelopes of limiting streamlines. Relevant review articles and discussions of important problems have been prepared by Topak and Peake (1979, 1982) and Peake and Topak (1980, 1983).

In most cases calculations have not been able to identify separation lines mainly because the numerical methods used in three-dimensional boundary-layer flows have not been sufficiently accurate to represent the zones of influence and dependence governing the three-dimensional boundary-layer development along the surface. Early investigations of the separation problem were reported by Wang (1974a) who paid careful attention to the zones of influence and dependence in his formulation of the computational scheme. These calculations were performed for a prolate spheroid with a thickness of 1/4 at angles of attack up to 30 degrees. The flow outside the boundary layer was assumed to be given by the potential flow solution and the results showed that the separation line at low angles of attack moves first forward and then recedes as one proceeds from the windward side to the leeward side, forming a pronounced tongue which points toward the nose of the body. To determine the location of the separation line, calculations were marched from both the windward and leeward planes of symmetry toward the center. At higher angles of attack, Wang was able only to calculate the separation line when advancing from the windward side and the lack of solutions from the leeward side made him conclude that the separation was of the open type, implying that the separation line was not connecting to the leeward line of symmetry. The separation pattern changed at about α = 42 degrees where the windward separation line joined the incipient separation near the nose.

Intrigued by the unresolved problems, Cebeci et al. (1981) repeated the calculations of Wang with a more sophisticated numerical method and solved the correct equations near the nose singularity which may account for some of the differences displayed. At α = 3 degrees, agreement between the two calculations is good and at α = 6 degrees, there is qualitative agreement but the calculated separation lines form an almost wedge-like intersection in the Cebeci et al. calculation. After careful study of the results, it was concluded that the flow behaves as if a singularity of the type postulated by Brown (1965) occurs as the windward separation line is approached. In other words, the flow seemed to behave as would be expected for a separation line formed by the envelope of limiting streamlines, but it should be remembered that singularities are difficult to detect numerically. Calculations started from the leeside came to an end in a peculiar manner in that as meridional shear stress component became negative, the crossflow shear stress increased and the displacement thickness stayed almost constant while the solutions did not converge or fail because information was not available from the previous x-station. Although the singularity-type behavior of the solutions could not be conclusively extracted, separation was evident. At higher angles of attack (α = 15 and 30 degrees), the solutions approaching the windward separation line behaved as before except that the zero shear stress line for the crossflow was now close to the separation line. Calculations started from the leeside invariably broke down when the calculations should have made use of information which had to come over the separated region. It was inferred that the lower boundary or limit for this condition is given by the location of the inviscid streamline which crosses above the most forward point of the windward separation line. This streamline is called the accessibility

line. Referring back to the α = 6 degree case, it is now clear that the accessibility and leeward separation lines are essentially identical.

Since there are regions which are not accessible for calculations, the assertion of an open separation does not apply to prolate spheroids when the external pressure is prescribed. It is also clear that the calculated windward separation line is for all practical purposes an envelope of limiting streamlines for α = 15 and 30 degrees, the proof being that the line is very close to the crossflow reversal. Calculations with interactions still remain to be done, and it is anticipated that such a calculation will show the separation line to be a limiting streamline. This interpretation is based on the observations that open separation exists in experiments and that open separation is not possible when calculations were performed without interaction, as proved by Cebeci et al.

3.0 APPRAISAL OF CURRENT KNOWLEDGE AND ABILITIES

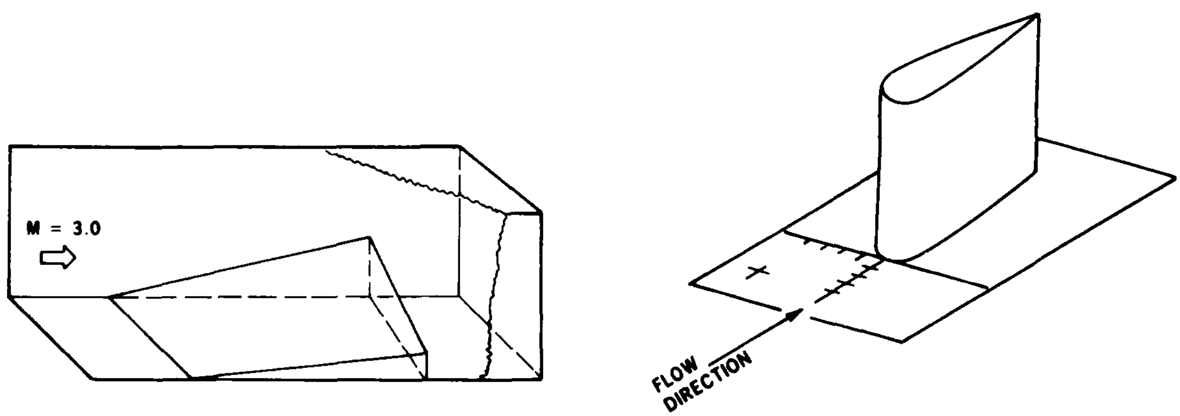
3.1 Experimental Work

Experimental results are essential to the development and evaluation of calculation methods and it is useful to review them and the understanding which they provide. It is convenient to refer to three-dimensional flows under the headings of wings, missiles and ships, since they have distinctive features, and this pattern is followed here even though measurements in the flow around ship hulls are few. In considering the flow around wings, some effects and simulations such as an airfoil projecting from a flat surface are considered.

Wing-Like Geometries

Three building-block experiments are being conducted under the sponsorship of NASA Ames to address the issues involved with three-dimensional turbulent boundary layers. The first two experiments are pressure-driven flows being studied by Pierce at Virginia Polytechnic Institute and by Demetriades at Montana State University; the third is a shear-driven flow at NASA Ames. The first two flows provide incompressible and compressible cases with crossflow determined by the model geometries. Demetriades experiments are for a compressible flow in which a "twisted wedge" model (see Fig. 8a) was designed to give a three-dimensional boundary layer at supersonic speeds with constant lateral pressure gradient with longitudinal gradients. Surface and flow-field data were documented in 1982 and computations began in 1983.

Pierce's studies correspond to incompressible flow in which the three-dimensional flow is generated by a teardrop body with axis normal to the flat-plate tunnel flow (see Fig. 8b). The experimental set-up is similar to that used earlier by East and Hoxey (1969) who measured mean-velocity profiles by a three-tube yaw probe in the three-dimensional boundary layer upstream of (I) and including the three-dimensional separation (II). Pierce's measurements, see Fig. 9, include these two regions plus the three-dimensional horseshoe or junction vortex system contained between the separation line and the body itself (III). No studies are planned for regions containing the near wake (IV) and the far wake (V).



(a) (b)

Figure 8. Experimental arrangements of (a) Demetriadis, (b) Pierce.

The flow in region I has been studied by Pierce and his associates for approximately four years, first to test 10 near-wall similarity models for three-dimensional turbulent boundary-layer flow (Pierce et al. 1983, 1983a,b) and subsequently to document a standard test case, including measured upstream and edge conditions, together with mean velocity and turbulent stress field measurements in the three-dimensional pressure-driven boundary-layer-like flow field, but not including the separation region. These data, which can be used to validate computational models and predictions for such a flow and are to be released soon, include mean velocity and turbulent stresses (all six components) on an upstream initial condition plane, local freestream mean velocity or edge conditions from the upstream initial condition plane through the downstream flow itself, wall pressure-field measurements in the downstream flow, direct force local wall shear stress measurements of the downstream flow and mean velocity and turbulent stresses (all six components) in the downstream flow.

A portion of the work corresponding to region II has been accomplished (Pierce and McAllister, 1982) and other portions such as documenting the mean flow structure in the three-dimensional separation region and locating the mean position of the separation sheet/envelope are in progress. The work for region III, which will involve both direct and indirect local wall shear measurements, has not yet started.

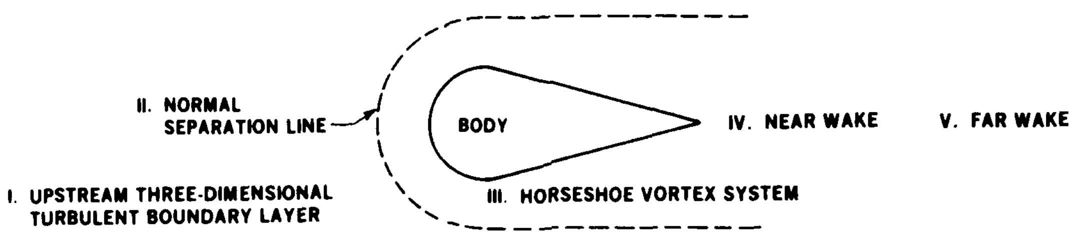


Figure 9. Regions of flow identified by Pierce.

The shear-driven flow which is generated by a spinning cylinder (Fig. 10) provides guidance for modeling the pressure-strain correlations and the low Reynolds number terms used in various turbulence models. The test zone is within the boundary layer on the fixed cylinder downstream of the spinning section, where the rate of return to axisymmetric flow is solely a measure of the viscous stresses acting on the fluid. Accomplishments so far include skin-friction measurements using several techniques, comparisons of data with best available turbulence models and a preliminary arrangement for three-dimensional laser-Doppler velocimeter (LDV) measurements. Goals include making LDV measurements, assessing the improved turbulence modeling and attempting to impose a pressure gradient on the flow to study its effect on modeling coefficients.

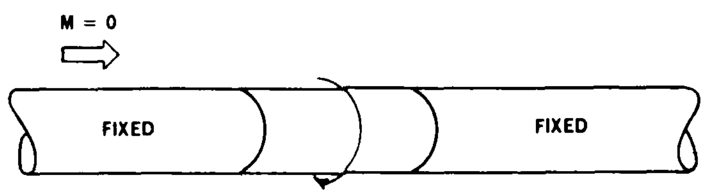


Figure 10. Experimental arrangements of the shear-drive flow of NASA, Ames Research Center.

Until recently, most experimental data pertaining to the growth of a turbulent boundary layer on a swept wing were obtained in low-speed flows and, except for the measurements of Lux (1978), no data existed for transonic flows. Now, thanks to Lockman and Seegmiller (1983), Spaid and Roos (1983), and Spaid (1984), some information, albeit limited in scope, does exist. Lockman and Seegmiller (1983) made measurements in turbulent subcritical and supercritical flow over a swept NACA 0012 semispan wing with an aspect ratio of 3, in a solid-wall wind tunnel. The measurements were obtained for a range of freestream Mach numbers (0.5 to 0.84), Reynolds numbers based on wing chord (2×10^6 to 8×10^6), and angles of attack (0 to 2°) and provide a variety of test cases for assessment of wing computer codes and tunnel-wall-interference effects. The supercritical cases include flows both without and with three-dimensional flow separation. The principal results were mean surface pressures for both the wing and tunnel walls. In addition, surface oil-flow patterns on the wing and mean-velocity, flow-field (by LDV) were obtained for supercritical flow. All the pressure and flow-field data are given in tabular form, with representative results presented graphically to illustrate some of the effects of the test parameters. Comparisons of the wing pressure data with results from two inviscid wing codes are also shown to assess the importance of viscous-flow and tunnel-wall effects.

In order to create a three-dimensional boundary-layer flow in which its interaction with the wind-tunnel boundaries is minimum, Mateer and Bertelrud (1983) conducted tests on a swept wing of constant chord (21 cm) that spanned a fully contoured test section nominally of 25 x 38 cm; that is, all four walls of the rectangular test section were shaped to conform to the inviscid streamlines. The wall contours were obtained by calculating the inviscid flowfield around a wing having a sweep of 32 degrees and NACA 0012 section streamwise. Surface pressures have been measured on the wing and tunnel walls for a range of Mach and Reynolds numbers, and the influence of the tunnel walls assessed from comparisons of measured results and inviscid computations. Oil-flow patterns on the wing also have been obtained and compared with streamlines computed from a viscous code using various turbulence models. Although their study is not yet complete, the data summarized in their paper is useful in indicating the reduction of the effects of wind tunnel walls on the flow over a swept wing by wall contouring.

The measurements of Spaid and Roos were conducted in the NACA/ARC 14-foot wind tunnel on a 1.113m semispan transport model with an aspect ratio of 6.8 and sweep angle of 35°. The data include surface static pressure and boundary-layer profiles (velocity magnitude and flow direction profiles) obtained by three-orifice yaw probes from about $x/c = 0.3$ to 1.0 at nine spanwise stations, see Fig. 11. The main tests were conducted at the design cruise conditions of this wing with values of Mach number and lift-coefficient corresponding to $M_\infty = 0.825$, $C_L = 0.523$ and a Reynolds number based on the 0.359m mean aerodynamic chord (R_c), of 4.5×10^6. Tests were also conducted at $M_\infty = 0.5$, $C_L = 0.583$ and $R_C = 3.6 \times 10^6$. Transition was fixed at $x/c = 0.06$ on both the upper and lower surfaces. The results show an upper surface shock wave near midchord over most of the span; the shock-wave weakens and moves forward near the tip as a result of the wing twist. There is some evidence of probe interference effects and significant flow unsteadiness at the higher Mach number; low frequency disturbances due to vorticity caused the shock to oscillate.

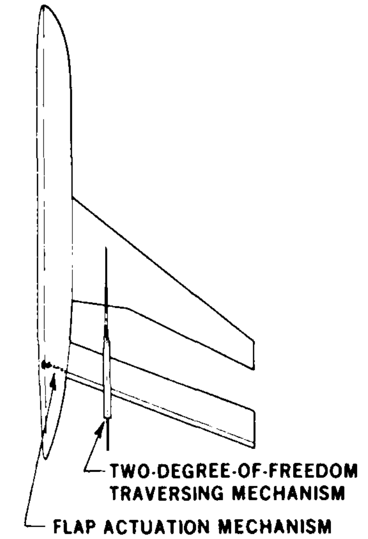


Figure 11. Wing, fuselage, and boundary-layer traversing unit of Spaid and Roos (1983).

Some problems of the blades of axial compressors and turbines are common to those of wings. In particular, the corner vortex associated with a wing-fuselage intersection is similar to that which occurs at the junction of a blade and an end wall although, in the latter case, the stronger curvature of the blade and the confinement associated with the blade passage may cause the vortex to grow more rapidly. Detailed measurements of pressure and flow properties have been obtained in stationary cascades, for example by Chima and Stragiser (1983) and Gaugler and Russell (1984) and provide additional physical knowledge of those pressure-driven vortices. In addition, heat and mass transfer measurements such as those of Goldstein and Karni (1984) help to explain and quantify some aspects of corner flows.These measurements are not considered in detail here because of the differences between wings and blades, but they can provide useful additional information.

Missile-Like Geometries

Some of the experiments on wing-like geometries are relevant to flow on bodies of revolution at incidence. For example, Hornung and Joubert (1963), East and Hoxey (1969), Dechow and Felsch (1977), Krogstad (1979), and Pierce and McAllister (1980) have considered the boundary layer ahead of symmetrical obstacles mounted on flat surfaces, with flow separating just ahead of the obstacle. Experiments on prolate spheroids at low speeds are directly relevant to missiles and include mean-velocity results of Choi (1978) and the more extensive investigations of Meier and Kreplin (1980), Kreplin et al. (1982), Meier et al. (1983) and Meier et al. (1984a,b) in the windward and leeward planes of symmetry of bodies of revolution. No turbulence data were available until the recent experiments of Baek (1984) whose experimental results consist of (a) the pressure distributions, (b) the distributions of the mean velocity components u and w measured by the yaw probe, and u,v,w by the hot-wire probes, (c) the Reynolds stresses, $\overline{u'^2}$, $\overline{v'^2}$, $\overline{w'^2}$, $\overline{u'v'}$ and $\overline{u'w'}$, and (d) the triple products $\overline{u'^3}$, $\overline{v'^3}$, $\overline{u'v'^2}$, $\overline{u'^2v'}$, $\overline{w'^3}$, $\overline{u'w'^2}$ and $\overline{w'u'^2}$. The complete set of data has been tabulated in Appendices A, B and C in Baek's thesis and correspond to windward and leeward sides of a prolate spheroid with an angle of attack of 15°.

In order to accurately predict the performance of aerodynamic inlets, heat transfer on reentry vehicles or heating of high-speed tactical missiles flying at an angle of attack, it is necessary to have some prior knowledge of the three-dimensional boundary layer on the configuration. For this reason, Yanta et al. (1982), Yanta and Ausherman (1983) and Ausherman et al. (1983) have conducted experiments to investigate the flowfield about a sharp cone in supersonic flow to provide a database for use in conjunction with computer codes which will predict the flowfield over a conical body at small angles of attack. These studies were made with a sharp cone 7°-semi-vertex angle for angles of attack of 0, 2 and 4 degrees at a freestream Reynolds number of 7×10^5/m and Mach number of 3.0. The measurements included mean velocity components, turbulence intensities and Reynolds stresses for seven circumferential locations around the body and one axial station and were obtained with a three-dimensional Laser Doppler Velocimeter (LDV). Both the viscous and inviscid portions of the flowfield were investigated, yielding information about the velocity profile behavior in both regions. In addition, skin-friction measurements were made with a surface mounted Preston tube at five axial stations along the cone. The velocity and wall shear-stress measurements were repeated, see Ausherman et al. (1983), for the cone modified by the addition of two different nose tips, a nominal 22-percent spherically blunted nose and a severely indented nose tip which represented an ablated reentry shape. In addition, the cone with different nose tips was also tested at $M_\infty = 5$, corresponding to a unit Reynolds number of 2.6×10^7/m, the other test parameters being unaltered.

In a recent paper, Peake et al. (1982) investigated the three-dimensional leeward separation about a 5-degree semiangle cone at 11 degree incidence in the Mach number range from 0.6 to 1.8. The surface pressure and surface visualization measurements are complemented by numerical calculations based on the solutions of the Navier-Stokes equations. Useful surface flow visualization and LDV measurements of laminar flow profiles on a prolate spheroid were taken by Telionis and Costis (1983).

Ship-Like Geometries

Experimental information of thick three-dimensional turbulent boundary layers on bodies of revolution and ship forms has been reviewed by Patel (1982). Larsson (1974), Hoffman (1980), Hoffmann (1976), Nagamatsu (1980), Hatano and Hotta (1982) and Hayashita (1982) have all reported measurements of direct relevance to the flow around the hulls of ships and clearly demonstrate that the complicated curved shapes associated with the bow region and the thick boundary layers associated with the long lengths and the pressure gradient in the stern region are of major importance. The results of these investigations are mainly of the mean flow, but it is clear from the small number of turbulence measurements, that the inner region of the thick stern-region boundary layers are similar to those of thin boundary layers but that the outer region is characterized by unusually small stresses.

Because of the importance of the aft part of the hull and the unusual nature of the flow there, Ramaprian, Patel and Choi (1981) and Groves, Belt and Huang (1982), for example, conducted their experiments with a combination hemisphere, semi-spheroid body at 15 degree incidence and on an elongated body of 3:1 elliptic cross section at zero incidence, respectively. These mean-flow measurements confirm that the development of the thick boundary layers is the convergence of the external-flow streamlines in planes parallel to the surface for both the axisymmetric and three-dimensional flows. In the latter, it is also clear that the growth rate varies considerably around the ellipsoid cross section although the cross-flow angles were less than 5 degrees. The contributions of the David Taylor Naval Ship Research and Development Center have been considerable and include those of Huang and Cox (1977), Huang et al. (1976, 1978, 1980), Groves et al. (1982) and Huang et al. (1983, 1984) which concentrate on thick boundary-layer flows.

It is evident that the flow in the vicinity of the leading edge of a bulbous bow has much in common with that around the bodies of revolution of the previous section. The adverse and favorable pressure gradients which are encountered before the main part of the hull is reached are also associated with the geometry and again have features in common with bodies such as the prolate spheroids of Meier. The thick boundary layers of the stern region introduce a new feature in that the turbulence structure is different from that of thinner boundary layers and deserves special attention, even if this involves the approximation of rotational, potential flow. Further, and more detailed experimental work is clearly required, and is in progress at the David Taylor Ship Research and Development Center.

3.2 Computational Work

The computational work can also be conveniently described by identifying three categories, as in experimental work, according to the geometry of the configuration and each category can further be divided into two subcategories depending upon whether the boundary-layer equations are being solved for a prescribed external velocity distribution (standard procedure) or by an inverse procedure in which the external velocity distribution is computed as part of the solution. Until recently, most of the computational work in three-dimensional boundary layers has been limited to standard procedures; with the advances in our understanding of numerical procedures for such flows and with the need to interact viscous flow solutions with inviscid-flow solutions the interest in inverse boundary-layer procedures has increased as we discuss below.

The solution of the three-dimensional boundary-layer equations can be obtained by integral methods or by differential methods. While the differential methods can be applied to a wide range of flows with different boundary conditions, integral methods have the advantage that they require less computer time. For two-dimensional flows, simple wall boundary conditions, with appropriate choice of velocity profiles and auxiliary equations, the integral methods can give as accurate results as the differential methods. The methods due to Green et al. (1973) and Whitfield (1978) are typical examples of integral methods which have been shown to give accurate results for two-dimensional flows. Integral methods for three-dimensional boundary layers, on the other hand, are not as accurate as those for two-dimensional flows, even if the accuracy of the turbulence model is unimpaired, because of difficulties in parameterizing the cross-flow profile. Although the velocity profile expressions used successfully for two-dimensional flows can be generalized and extended to represent streamwise velocity profiles satisfactorily, it is the choice of cross-flow velocity profile which influences the accuracy of the integral methods. For example, Cebeci (1974) and Bradshaw (1971) reported calculations for low-speed turbulent flows with differential methods using eddy viscosity and one-equation stress model turbulence models, respectively. Among the flows they computed was an infinite swept wing on which Bradshaw and Terrell (1969) had made measurements over the rear, flat portion of the wing. The axial pressure gradient was nominally zero with a decaying crossflow. Comparisons of computed skin-friction, cross-flow angle distribution and wall streamline angle with both methods agreed well with the measured data. For this flow, calculations done by East (1975) and recently by Swafford (1983) show that the predictions of their integral methods are not as good as those of differential methods. East blames the poor performance of integral methods on the use of Johnston's cross-flow profile (1960) which is not representative of a decaying three-dimensional flow, particularly in the wall region. East also points out that this flow demonstrates that differential methods are generally more flexible than integral methods when applied to a variety of different types of flows. It is also quite likely that the performance of integral methods will not be as good as that of differential methods in three-dimensional flows with separation due to the rapid change of cross-flow velocity profiles.

Wing-Like Geometries

The computational activity in this area has concentrated on the development of three-dimensional boundary-layer methods for wings. Both integral and differential methods have been used. Yoshihara and Wai (1984) used an integral method in which the form factor relation is obtained from Green's planar lag-entrainment method (1973) and cross-flow related displacement and momentum thicknesses from Smith's three-dimensional integral boundary-layer method (1974). Street (1982) and Chow et al. (1984) use a version of the code originally developed by Dornier GmbH, which utilizes Stock's laminar method (1978) and Smith's turbulent boundary-layer method (1974) with some changes. Swafford's method (1983) is essentially an extension of Whitfield's two-dimensional integral method.

Among the differential methods, we see the same turbulence model (algebraic eddy viscosity) and with different numerical procedures and coordinate systems. For example, Kordulla (1978) used the Crank-Nicolson scheme with Krause's zig-zag procedure to overcome the difficulties in computing the flows with negative crossflow and McLean (1977) made modifications in his implicit finite-difference procedure for the same reason. While Kordulla and McLean both used an orthogonal coordinate system, Cebeci, Kaups and Ramsey, (CKR) (1977) used a nonorthogonal coordinate system and a numerical scheme which was not appropriate to flows with negative crossflow. A new numerical scheme, called the Characteristic Box, was devised by Cebeci and Stewartson (1978) and was successfully incorporated into the CKR code which also contained a geometry program to represent the wing analytically and to compute geometric parameters such as metric coefficients and geodesic curvatures. A typical computation time of this code for a typical transport wing is approximately 1 to 2 minutes on the IBM 3081 Model K.

The above methods use the standard approach and solve the boundary-layer equations for a given pressure distribution. If separation is encountered, the calculations come to a screeching halt due to the singular nature of the equations.

Some of the above boundary-layer methods have been interacted with inviscid codes for both subsonic and transonic flows. For the latter, several FLO-codes developed by Jameson and Caughey, were used and all solve the full-potential equation, but in different forms or in different coordinate systems. Among these is the FLO-22 code which is for a "wing alone" and uses a C-mesh produced by a parabolic mapping function. The full-potential equation is written in nonconservative finite-difference form, using Jameson's "locally rotated" operators. As with all potential equation methods, good approximation is expected only in flows with weak shocks. Also, the nonconservative formulation produces solutions which do not maintain

conservation through a shock and the effect of the nonconservative form is usually seen in an incorrectly predicted shock position.

FLO-27, FLO-28 and FLO-30 use a finite-volume scheme to produce conservative differencing of the full-potential equation. While FLO-27 is a wing and cylinder code, FLO-28 and FLO-30 are more general wing-body analysis codes which differ in the grids used to solve the flow equations. While FLO-28 does not map the crown-line into a coordinate line, FLO-30 does, and thus avoids the undesirable oscillations in pressure distributions along the crown-line. However, the improved mapping in FLO-30 introduces singularities upstream of the fuselage nose and results in distorted grids in that region. Since both FLO-28 and FLO-30 have shortcomings due to their grid generation scheme, several improved grid techniques have been developed. Improved versions of FLO-28 were developed by Yu (1980) and Chen, Caughey and Verhoff (1982) by mapping the crownline to the coordinate line. Yu (1980) modified the numerical grid-generation method of Thompson et al. (1974) to generate a wing-body grid system. While his method is capable of treating essentially arbitrary geometries, in many cases the source terms required to control mesh spacing introduced nonorthogonality into the grid system. An alternate, numerical grid-generation was later developed by Chen, Caughey and Verhoff (1982) to generate nearly orthogonal grids. An improved version of FLO-30 was developed by Chen, Vassberg and Peavey (1984) by generating a better grid without the singular line upstream of the nose.

Except for the study of Wigton and Yoshihara (1983), most of the interaction studies using the FLO codes have been conducted by direct boundary-layer methods. For a pressure distribution computed by one of these codes, solutions of the boundary-layer equations are obtained up to the separation line, determined either by a sudden increase in shape factor (integral method) or by the vanishing of the wall shear stress in the streamwise direction (differential method). Of course, none of these signals for separation correspond to a true definition of flow separation but are chosen because of the limitations of the direct boundary-layer procedure and of the numerical procedure. In the separated region, ad hoc assumptions are made to the displacement thickness distribution so that the displacement thickness distribution computed by the boundary-layer method and by this approximate procedure can be added to the basic wing shape in the surface normal direction. This technique was chosen by Yoshihara and Wai (1984), Street (1982) and by Chow et al. (1984) in preference to using surface transpiration boundary conditions, because it was easier to incorporate in the FLO codes. Yoshihara and Wai (1984) used FLO-28; most of Street's results were obtained with FLO-30; and Chow et al. used FLO-47, which is a modified version of FLO-27. All studies show, as expected, the strong influence of viscosity on the location of the shock wave and on the pressure distribution; the interaction schemes, however, break down in regions of flow separation and the effect of assumed displacement thickness distribution on the results is not clear. For example, Yoshihara and Wai's studies deal with high subsonic Mach number flows over swept wings at full-scale Reynolds numbers where "bubble-type" shock-induced and/or rear separation occurs. Viscous effects vastly improved the inviscid flow predictions, however, there is still room for improvement.

The application of three-dimensional boundary layers to low-speed flows has not received the considerable attention assigned to transonic flows due to the strong influence which viscous effects have on the shock waves in transonic flows. Recently, pilot studies in low-speed flows have been conducted by Cebeci (1983) to explore the merits of his two-dimensional interaction scheme to three-dimensional flows. At first, simple geometries such as a clean wing and a wing and flap combination were considered with his two-dimensional inverse boundary-layer procedure used in a strip-theory sense. Figures 12 and 13 show the preliminary results for these cases. As can be seen from Figure 12, computed results agree well with experiment even though the boundary-layer calculations used the strip-theory approximation, which should be adequate as long as flow on the wing is not too three-dimensional. This is the case at small and moderate angles of attack when the flow separation is very small. However, at higher angles of attack, this approximation will become increasingly less accurate and will require the use of a full three-dimensional inverse boundary-layer procedure.

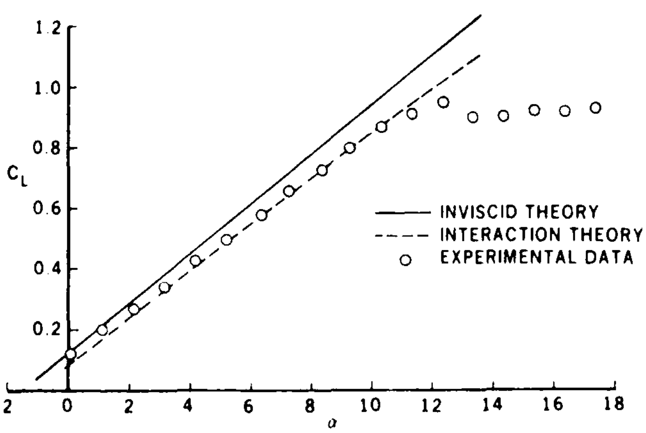


Figure 12 Comparison of results for a clean wing

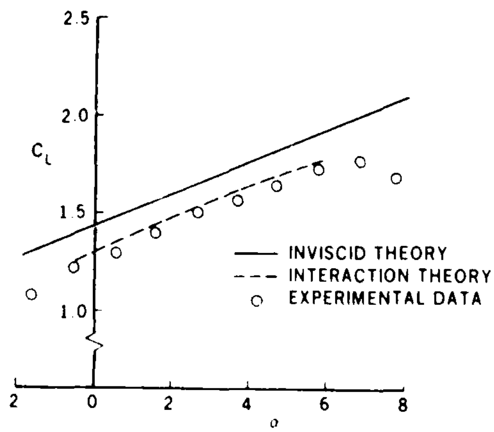


Figure 13 Comparison of results for a wing and 25° deflected flap

Missile-Like Geometries

Most of the recent computational work on missile-like geometries has been carried out in relati... body of revolution at incidence chosen to be a prolate spheroid with thickness t. This is a ... shape to study since analytical expressions are available for inviscid pressure distribution an... calculations are not required to determine the inviscid flow. It can be argued, however, ... calculation of boundary layers on such a body is more difficult than those on a wing ...

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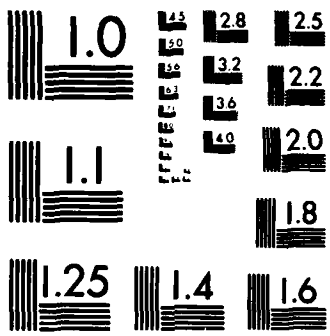

MICROCOPY RESOLUTION TEST CHART
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flow configuration which is strongly dependent on the angle of attack as discussed in Bradshaw et al. (1981). As a result, computations on such a body serve as an excellent test case to develop and evaluate numerical methods for three-dimensional flows and investigate the properties and the behavior of the solutions in regions of negative cross flow and in regions near separation.

There have been a number of calculations on this body in the past, particularly by Wang (1970, 1972, 1974a,b,c, 1975) who took careful note of the zones of dependence in formulating his numerical algorithm and used an implicit finite-difference method based on the Crank-Nicolson scheme. He was able to calculate the flow properties away from the separation line, but in regions of moderate or strong cross-flow reversals, he experienced difficulties due to deficiencies in his computational procedure and was unable to comment usefully on the nature of the solutions near separation. Recent calculations by Cebeci, Stewartson and Khattab used the Characteristic Box scheme for the same flow and took account of the varying direction of the local streamlines from that of the limiting streamline at the body to the external streamline, including any overshoots, and enabled accurate solutions to continue further than had been possible up to that time. Their calculations also showed that the concept of "open" separation proposed by Wang (1974a), has no place in the theory of three-dimensional boundary layers on prolate spheroids when the pressure gradient is prescribed; once it is found, a singularity occurs at the ok of accessibility.

The numerical method of Cebeci et al. developed for a prolate spheroid has been tested for both laminar and turbulent flows and the results have been compared with the experimental data of Meier et al. The results are shown in Figure 14 for laminar flow and in Figures 15 and 16 for laminar and turbulent flows. In performing the turbulent flow calculations of Figures 15 and 16, the algebraic eddy-viscosity formulation of Cebeci-Smith was used and the experimental transition distribution was specified to start the turbulent-flow calculations. We note from the comparisons that as long as we are away from flow separation, the agreement between calculated and experimental results is remarkable. As we approach separation, however, the agreement deteriorates since the inviscid pressure distribution differs considerably from the measured pressure distribution. Similar calculations up to flow separation were also performed by Patel and Baek for Meier's data with their ADI scheme and a two-equation turbulence model (k-ε model) and good agreement was obtained with experiment.

The recent data of Meier et al. (1984a) also contained velocity profiles which were obtained at a location 0.65 times the major axis from the nose, i.e. x/a = 0.30, and correspond to a freestream velocity of 55 m/s and to an angle of attack of 10 degrees. The measured and calculated results obtained by the method of Cebeci et al. are shown in Figure 17. Although the calculated boundary layer tends to be thinner at all circumferential stations, the agreement with experiment is good.

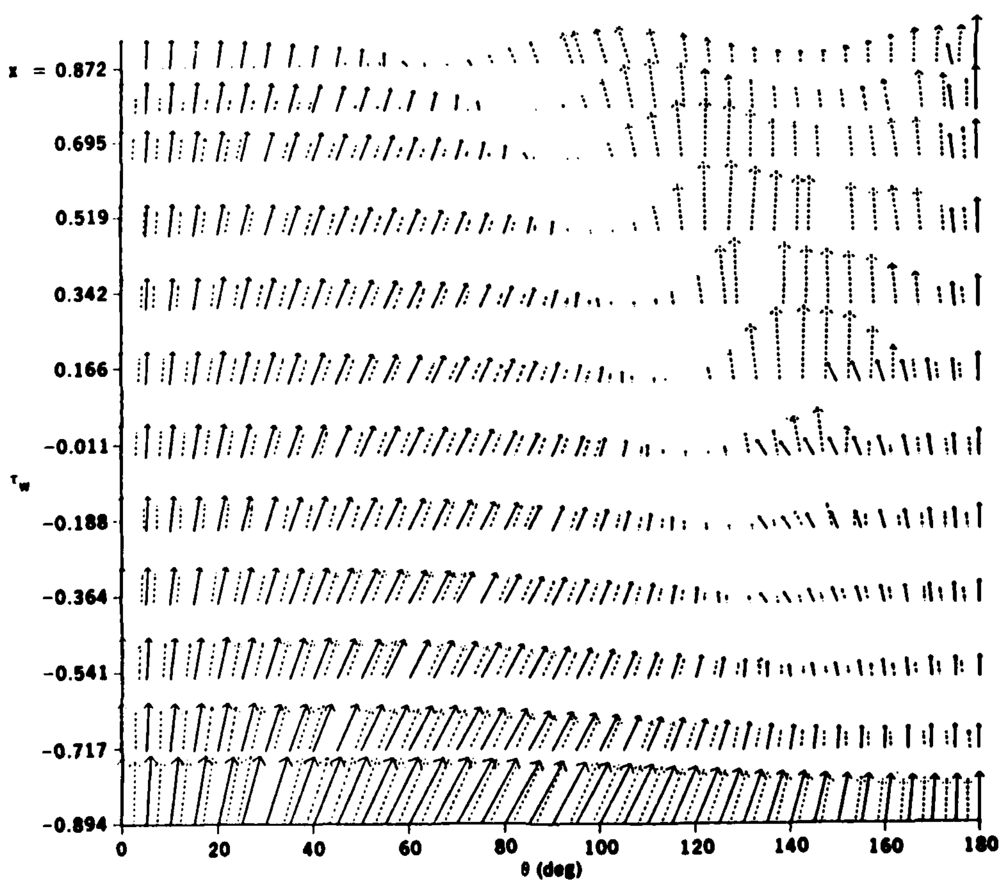

Figure 14. Computed laminar wall shear values on Meier's prolate spheroid as a function of axial distance at α = 10°. The solid lines denote calculations and the dashed lines the measurements.

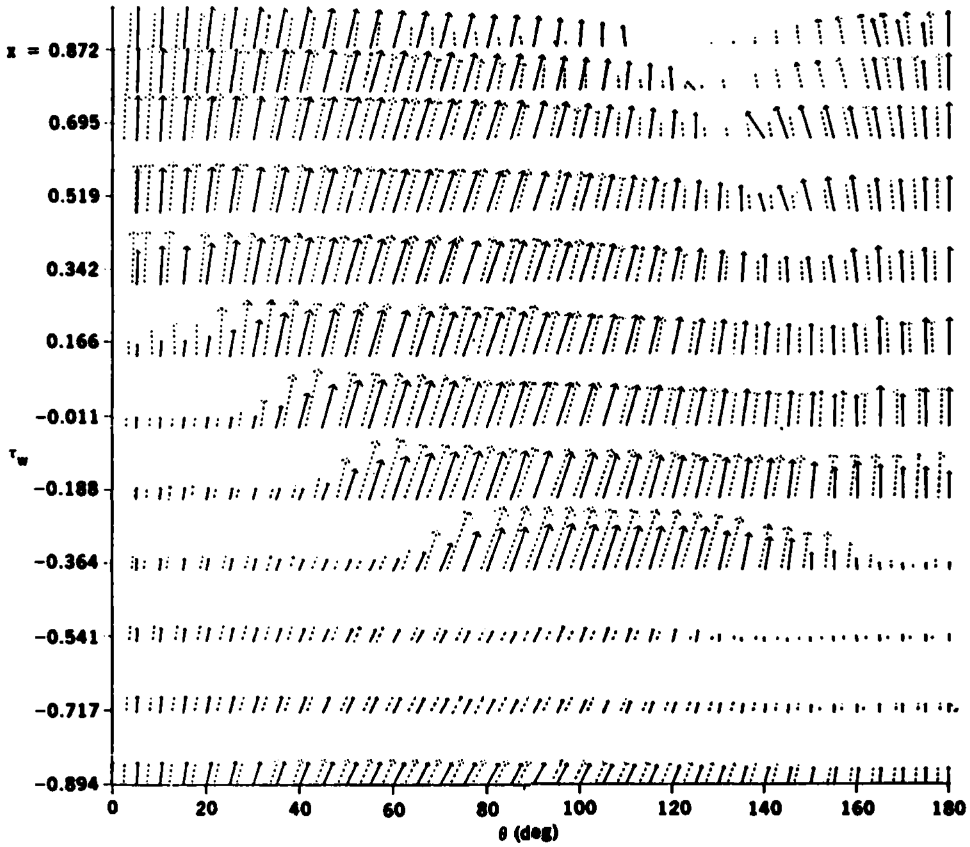

Figure 15. Computed laminar and turbulent wall shear values on Meier's prolate spheroid as a function of axial distance at α = 10°.

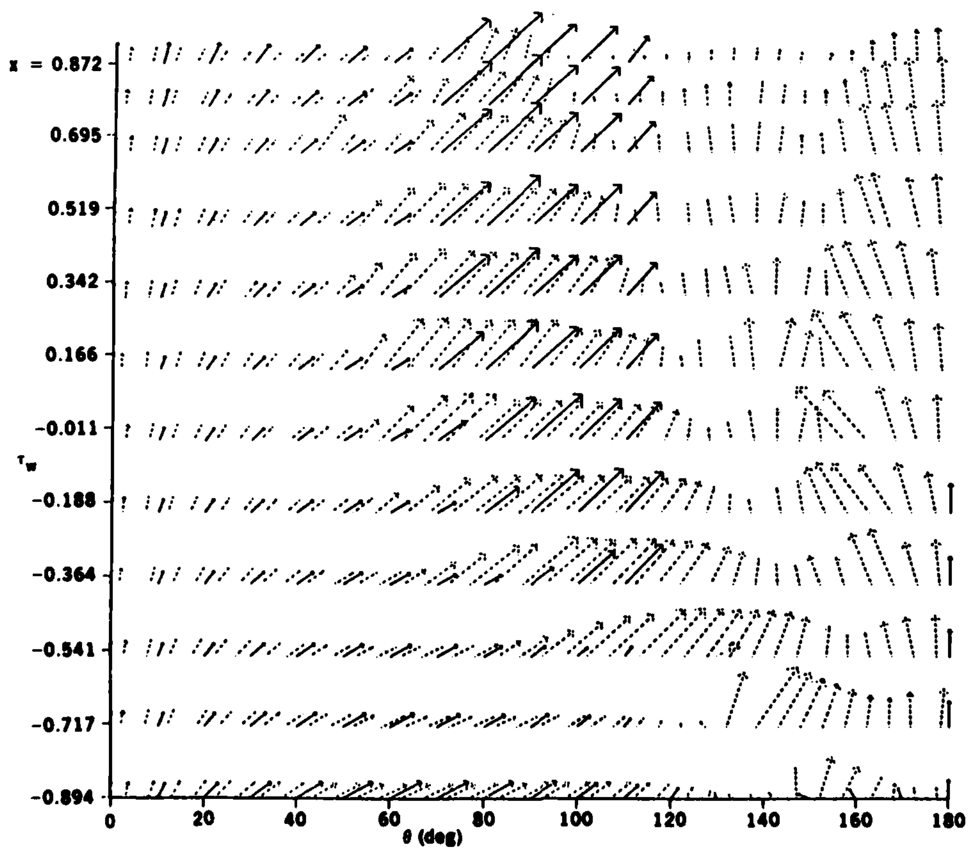

Figure 16. Computed laminar and turbulent wall shear values on Meier's prolate spheroid as a function of axial distance at α = 30°.

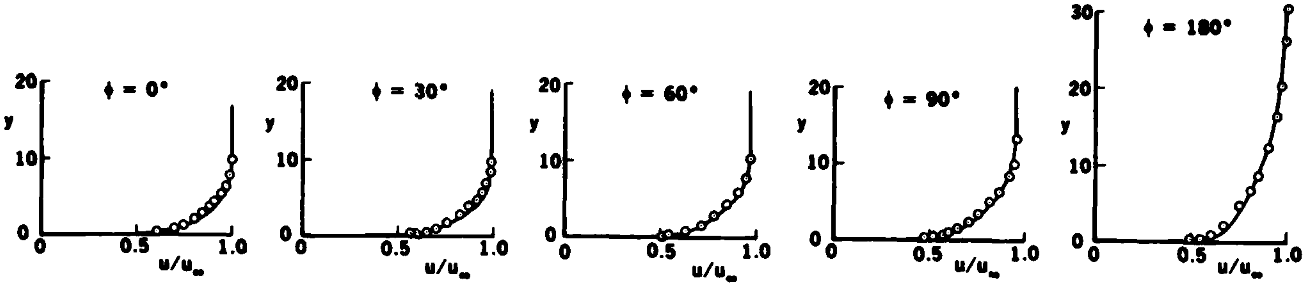

Figure 17. Comparison of calculated and measured turbulent velocity profiles for Meier's prolate spheroid at $\alpha = 10°$.

Ship-Like Geometries

Many propellers and appendages are located inside of ship stern boundary layers. Therefore, it is essential for naval designers to gain fundamental understanding and to obtain accurate prediction of this special class of external thick turbulent stern flow. The experiments at the David W. Taylor Naval Ship Research and Development Center confirm that the displacement body concept is an accurate approach for computing viscous-inviscid stern flow interaction. The measured static pressure distributions on the body and across the entire boundary layers were predicted by the displacement body method to an accuracy within one percent of dynamic pressure.

Neither the measured values of eddy viscosity nor the mixing length were found to be proportional to the local displacement thickness or other local boundary-layer thicknesses of the thick axisymmetric boundary layer. The measured mixing length of the thick axisymmetric stern boundary layer was found to be proportional to the square root of the local cross-section area of the turbulence region. This simple similarity hypothesis for the mixing length and the displacement body concept have been incorporated in to the Cebeci-Smith differential boundary-layer method by Wang and Huang (1979). The modified method predicts satisfactorily the measured mean velocity distributions in the entire stern boundary-layer flows of five sets of experiments having wide variation of axisymmetric body shapes and has been used as a reliable design tool. This simple modification of the Cebeci-Smith computer program yielded essentially the same results as those obtained from the solution of the partially parabolized Reynolds-Average Navier-Stokes equations for turbulent axisymmetric flows using flow streamline coordinates and the k-ε turbulence model (Hogan, 1984).

Theoretical and experimental investigations have also been performed to treat the interaction between the thick stern boundary layer and propeller (Huang and Groves, 1981). The hydrodynamic interaction between the stern boundary layer and the propeller was found to be inviscid in nature and calculations, based on an inviscid propeller/stern boundary-layer interaction theory, provide valuable insight into the manner in which the propeller interacts with a thick stern boundary layer.

4.0 CONCLUSIONS AND RECOMMENDATIONS

It is evident from the previous section that our knowledge of three-dimensional flows is incomplete and that our ability to make a priori calculations of the properties of the flow on complex geometrical configurations is far from perfect. It would be odd if it were otherwise in view of the conclusions of Cebeci et al. (1984) for two-dimensional flows. Although we can learn a great deal from the imperfect state of knowledge of two-dimensional flows, our knowledge of three-dimensional flows is much less and our calculation abilities are very much less mature. The clear implication is that additional experimental work is necessary and it is best that it be performed in close collaboration with the development of calculation methods which must also be the subject of carefully constructed research programs.

The following paragraphs consider ideas which may be of particular value in the formulation of future research. They are described separately for computational and experimental research and in that order. The first consideration of the calculation work reflects the assistance which the solution of conservation equations can give to the experimental work and the arguable view that the available tools of computational research need some priority in their development. This approach is also taken in the three subsections devoted to the three geometrical configurations singled out for special attention in the previous two sections.

It is useful first to consider what may be achieved by the solution of Reynolds-averaged forms of the Navier-Stokes equations and it should be recognized that the solution of these equations has only been possible in the last fifteen years and, to credible accuracy even for comparatively simple geometries, in the last five. Indeed, present arguments about the role of numerical accuracy, the relative merits of different forms of upwind differencing and attempts to compare solutions intended with, say, grids of 20 x 20 and 30 x 30 suggests that serious errors can occur even in the calculation of simpler flows. It is, therefore, evident that solutions of the Navier-Stokes equations are only likely to assist the design of missiles or transonic wings if they are applied to limited regions of flow. Research into the methods of solving these equations must continue and with increasing effort and care. One approach which is likely to be essential to the solution of the Navier-Stokes equations is to use the solutions of reduced forms of the Navier-Stokes equations as initial values so that the Navier-Stokes solver is asked to refine an approximate solution rather than to obtain an answer with no prior knowledge. The preferred reduced form is likely to vary with the flow under consideration and, in some cases, it could be that a hierarchy of several sets of equations would be formulated, and solved in sequence so as to asymptote to the correct result.

Historically, difficulties with the Navier-Stokes equations in particular and viscous-flow equations in general, precipitated the use of the inviscid flow equations which are still the mainstay of the design

of, for example, fighter airplanes and missiles. For low-speed flows, inviscid methods such as that of Hess and Smith (1964) and Hess (1974) have been in use for some time and are widely used in Industry, Government Laboratories and Universities. The methods of Bristow (1977) and the PANAIR code are worthy of special note. Similarly, for transonic flows, and due largely to the efforts of workers, including Ballhaus, Jameson, Holst, Hafez, Cheng, Caughey and Chen (see for example, Hafez and Cheng (1977), Hafez and Lovell (1984), Ballhaus et al. (1978), Holst and Ballhaus (1979), Holst (1980), Cheng (1982), Cheng et al. (1981), Ballhaus (1976), Jameson and Caughey (1977), Jameson (1979), Caughey and Jameson (1979), Caughey (1983), Chen (1983, 1984), and Chen et al. (1984)), considerable progress has been made towards appropriate and efficient solution methods for inviscid-flow equations. Most of them make use of the full potential equations but the successful solution of the Euler equation is within our grasp if concerted efforts are made.

The Navier-Stokes and potential-flow equations may be regarded as two extremes, one which recognizes the complete role of pressure, inertial and viscous forces and the other completely neglecting viscous effects. Several forms of equations may be solved between these two extremes and offer possible advantages which have not been fully explored. The interaction of solutions of potential-flow and Navier-Stokes equations was suggested above and has been considered, for example, by McDonald and Briley (1984). Where an attached boundary layer can be anticipated, interaction between potential-flow and boundary-layer equations is clearly to be preferred since the boundary-layer equations are much more amenable to solution than the Navier-Stokes equations and are known to represent thin shear layers. This clarity tends to disappear as the longitudinal pressure gradient imposed on the boundary layer becomes increasingly strong and adverse. Where separation occurs, it is necessary to interact solutions of potential-flow and boundary-layer equations in inverse form and, even with small regions of separated flow, the role of normal pressure gradients and other terms in the normal momentum equation are not well known. This suggests that part of all of the normal momentum equation be considered and a first approximation could be to include the pressure gradient term and the component of the inertial forces represented by $v\ (\partial v/\partial y)$. An alternative approach is to consider the so-called thin Navier-Stokes equations which include all inertial and pressure terms but neglect diffusion except in the direction normal to the wall. This approach avoids the interaction of two sets of equations, is simpler to solve than the Navier-Stokes equations, especially since the equations are parabolic, but may not be appropriate as the size of the recirculation region increases or as longitudinal diffusion becomes more important.

The discussion of the following paragraphs concentrates on the interaction between potential-flow and boundary-layer equations because this seems to offer particular advantages for three-dimensional boundary-layer flows and can be used in conjunction with other approaches. It is evident, however, that the basic knowledge is not available to allow firm choices of equation sets for particular flow problems. On the one hand, we have insufficient physical information of the role of the terms in the normal momentum equation as the strength of an adverse pressure gradient increases and where separation is reached: on the other hand, the uncertainties associated with the numerical solution of the different forms of equations, and their interaction, are not well known. Research of a fairly basic type is needed to quantify and remove these uncertainties.

In two-dimensions, the interaction of solutions of inviscid-flow equations and boundary-layer equations has led to the correct representation of viscous effects for many flows where it had previously been impossible. The efforts to improve these two-dimensional methods needs to continue but can now be supplemented by three-dimensional interactive methods which initially will make use of low-speed inviscid procedures and the full potential equations but can progress towards the solution of the Euler equations. Procedures for the solution of the three-dimensional boundary-layer equations exist for flows with prescribed pressure distribution and need to be developed for inverse procedures: the latter are essential in interaction methods for the reasons stated in Section 2. The method of coupling the solutions of the two sets of equations is important to the accuracy and cost of the results and it, together with the inviscid and viscous-flow solvers, needs to be accurate and efficient. To deal with complex geometries, the grid generation scheme is also important, as demonstrated recently by Chen, Vassberg and Peavey (1984) who have found that, with a new grid topology, it is possible to obtain improved wing-fuselage definition, better grid distribution and accelerated solution convergence. In a modified version of the FLO-30 inviscid-flow code, these new grid improvements lead to a 30 $\sim$ 50% reduction in computer cost, in addition they extend the application of the method to more general and realistic wing-fuselage configurations.

The incentive for further development work can readily be made clear by reference to examples of costs of calculations performed with existing methods. Extensive experience with a Neumann code has been documented by Hess and Friedman (1981) who show that the airfoil section needs to be represented by some fifty panels if acceptable accuracy is to be achieved and that about ten such locations are required across the span. Thus, around 500 panels are needed for a clean wing. The computer time required to represent the inviscid flow around the wing configuration of Lovell (1977) was approximately 4 minutes of IBM 3081-K at a cost of around $200. The corresponding computer time for the solution of the boundary-layer equations using a strip-theory inverse method, due to Cebeci, was 3 minutes at a cost of $150 so that one cycle comprising an inviscid and viscous flow calculation required 7 minutes and $350. The experience of Cebeci (1983) is that some five such calculation cycles are required to achieve convergence, at a total cost of around $1750.

Similar tests to those of the previous paragraph, have been reported by Cebeci (1983) for the same wing with a trailing flap at 25 degrees deflection angle. In this case, one thousand panels were required and the time to solve the boundary-layer equations doubled so that the resulting cost of a single calculation cycle was around $1300. Five iterations again achieved convergence at a total cost of $6500 for one angle of attack. It should be pointed out that with flow separation and with a higher angle of attack, the use of the strip-theory boundary-layer scheme becomes less accurate and should be replaced by full three-dimensional boundary-layer calculations which will lead to approximately 20% addition computer time and cost. The preliminary studies of Cebeci also indicate that the calculations become, as expected, very costly as the complexity of the flow configuration increases. This is partly because the computational procedures which involve interaction are in the development stage and partly because the problems being tackled are complex, requiring large amounts of computer time even for purely inviscid flows. The

initial estimates indicate that the calculation of an inviscid flow for a wing-body combination with four elements will require 2000 panels costing nearly $4000 with a CPU-time of one hour. The boundary-layer calculations will require less, say 1/2 hour of computer time at a cost of $1000/cycle. Thus, the cost of one cycle will be 1-1/2 hours at $5000. If we assume, as in the wing-flap combination, that five cycles are needed to obtain a converged solution, then the computer time will be nearly 8 hours at $25,000. This, however, is for only one angle of attack and more angles of attack are needed to obtain the complete aerodynamic characteristics of the configuration. Although the inviscid/viscous approach appears to be costly, it is the most promising approach and greatly reduces the reliance on empirical methods currently used in aircraft design.

It might first be assumed that the solution of the boundary layers is cheap and that numerical developments are not urgently required. Again, iterations within an interactive scheme and a number of test cases rapidly imply that this is not so and improvements to the accuracy and economy of three-dimensional boundary-layer methods are increasingly important as their use for design purposes increases. Specific recommendations for future work include the following.

1. Develop noniterative methods for three-dimensional flows. The boundary-layer equations are nonlinear and Newton's method is often used to achieve linearization and quadratic convergence, which for laminar and turbulent flows requires no less than three iterations. In recent work, Cebeci (1983) has developed and applied noniterative methods for two-dimensional flows and has shown that they can reduce computer costs by more than 50% while obtaining the same accuracy. Work of this type needs to be extended to three-dimensional flows.

2. Develop and improve extrapolation methods from y^+ of around 50 to the wall, particularly for use with forms of the Navier-Stokes equations. Present eddy-viscosity formulations use zero slip conditions at the wall and make use of up to ten grid points in the near-wall region to $y^+ = 50$. This approach can again lead to significant savings of computer time for both boundary-layer and Navier-Stokes equations. Particularly in the near-wall regions of separation bubbles, better extrapolation methods are required.

3. Improve linearization methods for turbulent-diffusion terms. At present, differential methods which use the eddy-viscosity concept in algebraic form, apply Newton's method to all terms in the boundary-layer equations except for the terms which involve the eddy-viscosity formulas. In recent work, Cebeci, Chang and Mack (1984) have introduced a new linearization scheme which makes use of the Mechul-function method of Cebeci (1976) and tested it for two-dimensional flows. A sample of the results is shown on Figure 18 which clearly shows the advantages to be gained from this method. Its extension to three-dimensional flows is highly desirable.

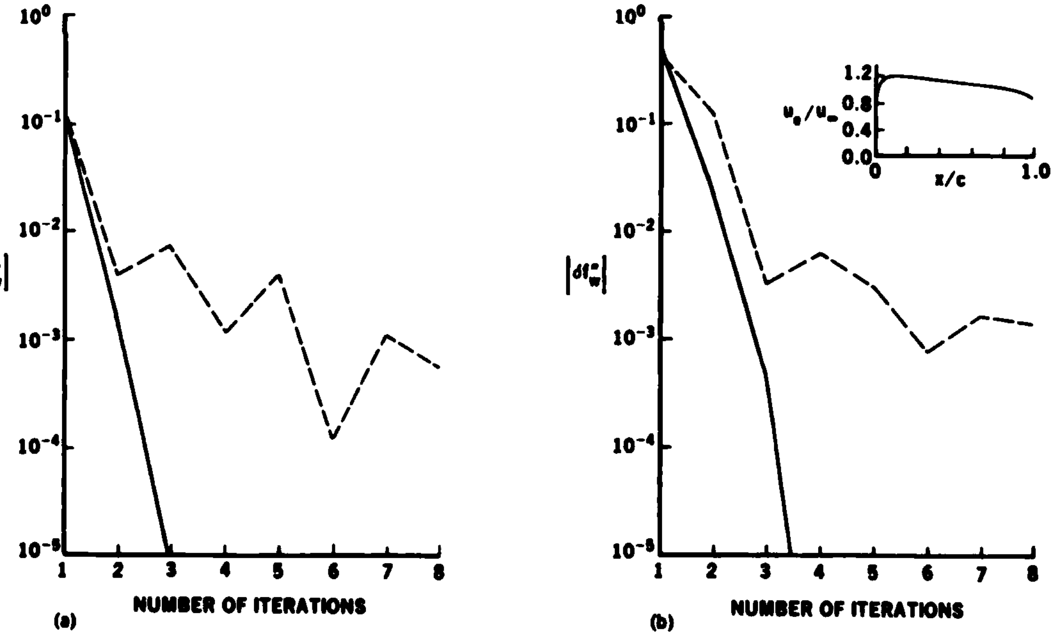

Figure 18. Rate of convergence of wall shear values for turbulent boundary layers with two linearization schemes. Solid lines denote full Newton (new scheme) and dashed incomplete Newton (old scheme). (a) Zero pressure gradient. (b) With pressure gradient at x/c = 1.0.

The improvement of interactive procedures, in addition to the improvement of the solution methods for the inviscid- and viscous-flow equations which they contain, require careful theoretical consideration. Physical knowledge of three-dimensional boundary layers is incomplete and the nature of three-dimensional flow separation is poorly understood at the present time. Well formulated experimental investigations are required to improve our physical knowledge, as discussed later in this section, and improvements to interactive procedures conducted in relation to existing information in the meantime. These improvements are the subject of the following paragraphs.

To describe some of the areas which require attention, a laminar incompressible flow over a body of revolution is considered. A first step in computing the flow properties for this configuration at high Reynolds number is to assume that the flow is largely attached and calculate the boundary layers on the assumption that the external velocity is given by inviscid flow theory. In the studies reported by

Cebeci, Khattab and Stewartson (1979, 1980, 1981) these calculations have been carried as far as it is mathematically and physically possible, but do not cover the whole of the spheroid. The accessible region includes the forward stagnation point and is bounded by the normals to the body through the separation line on the windward side, the external streamline passing above its most forward point or ok, and the leeward separation streamline if it occurs early enough. Figures 19 and 20 show two sketches of the surface streamline in laminar flow for angles of attack of 6° and 30°, respectively, and illustrate the accessible region. In these figures, x is the distance along the major axis from the nose and ϕ is the angular distance around the spheroid from the windward line of symmetry so that the leeward line of symmetry is given by $\phi = \pi$. There is still some uncertainty about the direction of these lines close to the (inferred) separation line on the leeward side of the body.

There is strong evidence from the numerical calculations of Figures 19 and 20 and from asymptotic analyses that on the windward separation line there is a singularity in the flow which terminates the computation there. On the leeside of the spheroid there are two properties which force the termination of the computation. First, it appears that the surface streamlines must turn through almost 180° to approach the separation line from the aft direction. The present numerical methods cannot cope with this phenomenon but fortunately it only develops within a degree or two of the separation line. Secondly, the flow is singular at the ok and the effect of the singularity is transmitted across the velocity profiles at this point. Further, it is convected with the local fluid velocity and so will cause the termination of the solution on the normals to the surface passing through any streamline originally lying above the ok. To an observer coming from upstream and assuming that he has not already encountered leeside separation, the first such streamline is the external streamline through the ok. Numerically, the phenomenon manifests itself by dramatic thickening of the boundary-layer profile, i.e. the shear at the edge of the boundary layer, up to that time is almost zero, suddenly becomes comparable with its value at the surface.

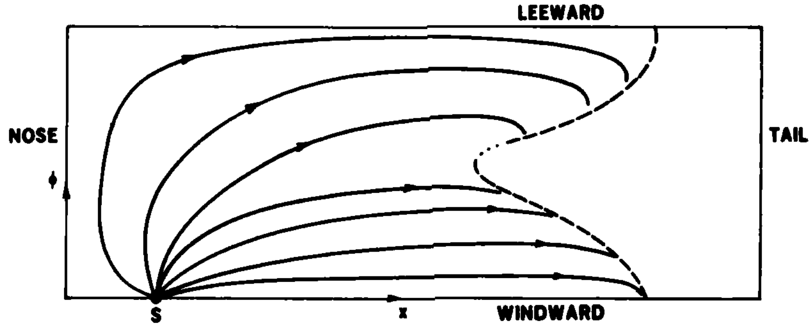

Figure 19. Sketch of surface streamlines in laminar flow on a prolate spheroid for $\alpha = 6°$.

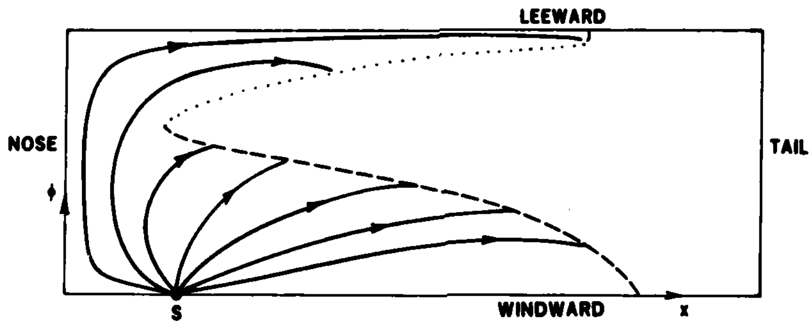

Figure 20. Sketch of surface streamlines in laminar flow on a prolate spheroid for $\alpha = 30°$.

Let us now discuss the physical significance of the properties of these boundary layers. In two-dimensional flows past bluff bodies, such as circular cylinders, the assumption of attached inviscid flow is clearly wrong, especially over the rear of the cylinder, and the associated boundary layer develops a singularity at separation. It is tempting to deduce that the two are linked so that a boundary-layer singularity implies that the assumed inviscid flow is incorrect. There is something in this argument but the linkage is not so strong that one must follow from the other. Consider the flow over an airfoil at incidence, as for example by Cebeci and Schimke (1983). Here separation occurs with a singularity if the external velocity is prescribed from attached inviscid theory. If, however, we permit the boundary layer to interact with the external flow, separation occurs without a singularity and so the calculations may be extended to include reversed flows. We may conclude, therefore, that the onset of separation does not itself signal the termination of the attached-flow assumption.

On the other hand, there is no question that the interactive boundary-layer theory with attached inviscid flow can be used in all cases of separated flow. Either it must break down by the separated region becoming larger and larger, both longitudinally and laterally, until the assumptions of boundary-layer theory fail or by a sudden change in the global structure of the flow from attached to detached. It is possible that the latter situation may develop independently of boundary-layer theory, in which case the interactive theory would cease to be relevant and without warning. There are some indications of this speculation for two-dimensional flows as, for example, in the interactive calculations of Cebeci (1983) performed in relation to NACA airfoils 4412 and 0012 and shown on Figures 21 and 22, respectively. As can be seen, the comparison with experiment is good up to the stall angle and slight discrepancies with data

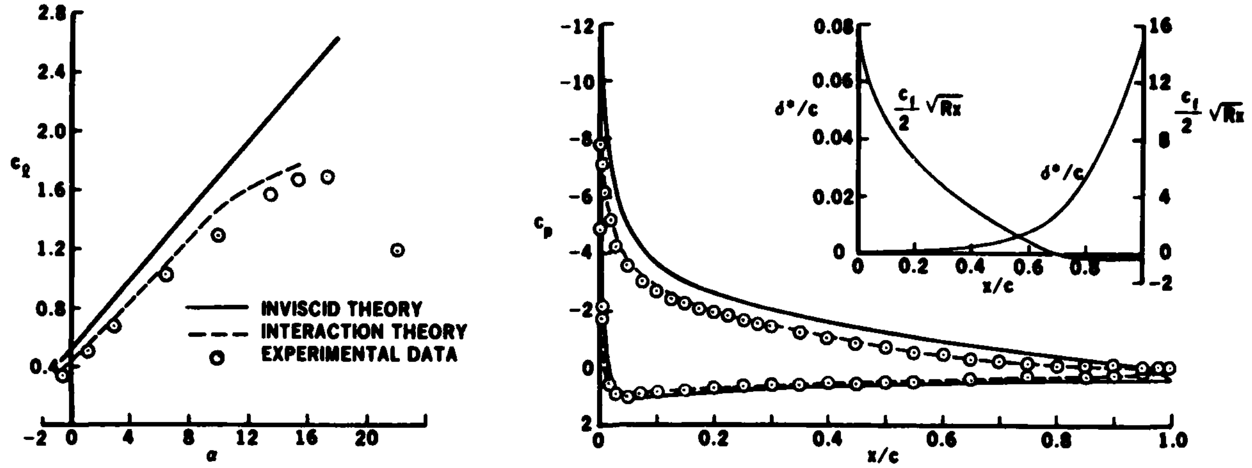

Figure 21. Comparison of results for the NACA 4412 airfoil.

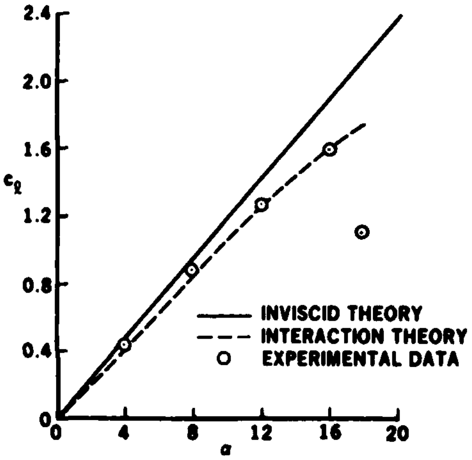

Figure 22. Comparison of results for the NACA 0012 airfoil.

can be due to the FLARE-approximation which becomes increasingly more important with large regions of recirculation and also due to wake effects which were neglected in the calculations. We also note that calculations were not carried out beyond $\alpha = 16°$ for the NACA 4412 airfoil; for the NACA 0012 airfoil they were carried out beyond $c_{\ell\,max}$, and the calculations were not aware of the fact that they were in the forbidden zone! These results, however, are preliminary and additional work is necessary.

In the case of a body of revolution at incidence, it could be that as the angle of attack α is increased, interactive theory holds for $\alpha < \alpha_c$ but not for $\alpha \geq \alpha_c$ even though it is intrinsically satisfactory for $\alpha > \alpha_c$ as well as $\alpha < \alpha_c$. This possibility is unlikely, and the relevance of interactive theory to the flow over a particular configuration can be decided on internal grounds of existence and consistency, i.e. if we can find a solution of the interactive equations in which the boundary layer remains thin, then this is a correct description of the global flow. Two recent studies lend support to this conjecture and also indicate that the limit of theory may, in certain circumstances, be signaled by the failure to obtain a solution. This phenomenon is referred to as the interactive theory going sour by Stewartson.

The first example concerns the phenomena of leading-edge bubbles on thin airfoils, Cebeci, Stewartson, Williams (1981) and Stewartson, Smith and Kaups (1982), in which the inviscid velocity distribution with ξ_0 corresponding to an angle of incidence is given by

$$u_e = \frac{\xi + \xi_0}{\sqrt{1 + \xi^2}}$$

Here interactive theory permits a limited amount of separated flow, as shown in Figs. 23 and 24, before it fails. A rational theory of high-Reynolds number laminar flow supports this conclusion and a likely interpretation of the theoretical results is that dramatic changes take place in the global flowfield at high angles of attack, in the region in which the interactive theory failed.

The second example is somewhat artificial, but is well-posed mathematically and solutions have been obtained using both the Navier-Stokes equations, Briley (1971), and interactive theory, Cebeci and Stewartson (1983b). Again the two methods agree well and ultimately each fails under approximately the same circumstances. The failure of the interactive theory corresponds to inability to achieve convergence and of the Navier-Stokes equations to a similar phenomenon due in turn, perhaps, to instability heralding transition or, and more likely, to invalid boundary conditions.

The above discussion indicates the need for extension of the investigation of interactive methods for two-dimensional flows and comparable studies for three-dimensional flows. Such studies are necessarily basic and are largely concerned with achieving mastery over the boundary layer in the post-separation region with some given external conditions. They should be conducted to determine if we can integrate beyond the accessibility line and particularly across the windward line of separation using some interactive model. The real barrier at accessibility is provided by the singularity on the windward line of separation and associated with this phenomenon is the property of blowing velocity $v_w \to \infty$. Hence the singularity can be avoided provided we keep v_w finite.

As indicated above, experiments are necessary to provide additional physical information and to support the development and evaluation of the calculation methods. Experiments of direct relevance to specific

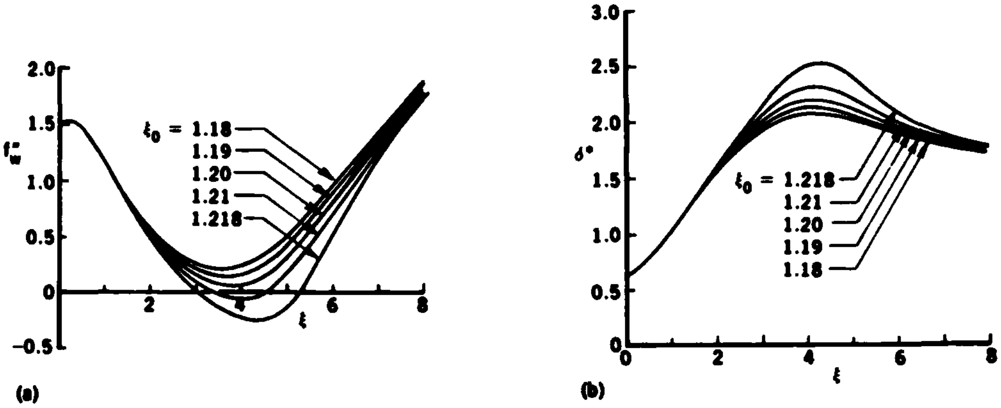

Figure 23. Results for a thin airfoil at various reduced angles of attack, ξ_0. (a) Wall shear parameter, f''_w. (b) Displacement thickness, δ^*.

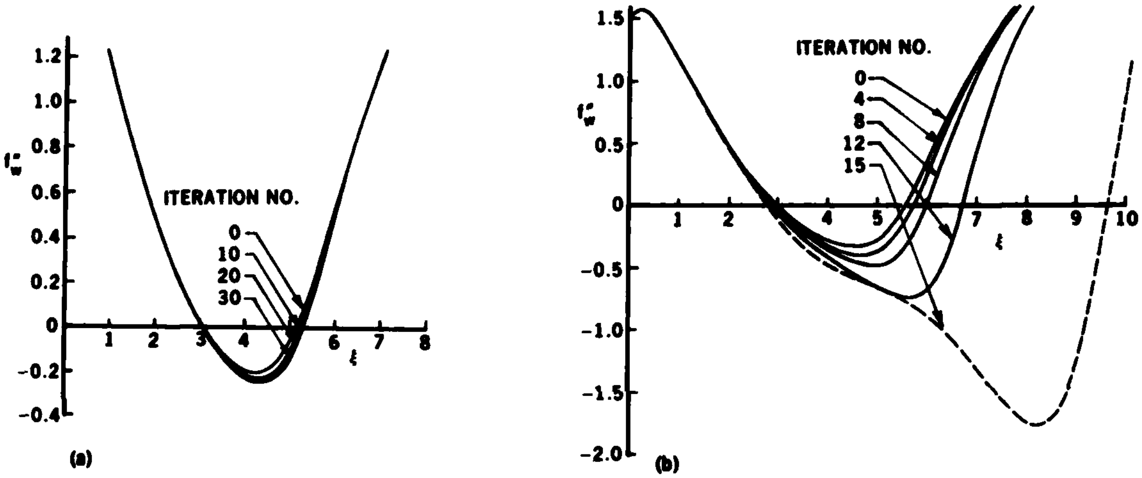

Figure 24. Variation of wall shear parameter f''_w with number of iterations. (a) ξ_0 = 1.218. (b) ξ_0 = 1.220.

wing, missile and ship configurations are few but are likely to grow in number as a consequence of the need to overcome particular problems. A few suggestions for this type of work are given in the following three sections. Here, the topic of experimental work is considered in relation to phenomenon which require special and immediate consideration. It is not the present purpose to formulate specific experimental plans but rather to draw attention to gaps which should be filled.

The topic of separation has been mentioned in various contexts in the previous pages and it is clear that the phenomena of separation and, where appropriate, reattachment are as important to three-dimensional flows as they are to two-dimensional flows. The onset of separation may occur in regions of laminar or turbulent flow, can cause rapid transition in the former case, and can give rise to separated-flow regions which are followed by reattachment (bubbles) or which can extend into a wake. Few experiments have been performed in the vicinity of separation partly because of the lack of appropriate instrumentation and partly because of the difficulties associated with the arrangement of a flow with the required flow characteristics. Exceptions include the work of Gault (1955) and Meier et al. (1984a,b) in which sophisticated instrumentation was not required to produce results useful for present purposes. Similar experiments on wings and on bodies of revolution are desirable even if limited to the specification of separation and reattachment lines for known geometry and flow boundary conditions. The use of thin-film gages, see for example Emrich (1981), can improve the accuracy of surface measurements but much can be achieved with high-quality flow visualization.

In recent years, the standard velocity measuring technique of pressure probes and hot-wire anemometry have been supplemented and complemented by pulse-wire, flying-wire and laser anemometry. These techniques allow, at least in principle, measurements within regions of flow separation and the results of Simpson et al. (1981), Nakayama (1984), Adair et al. (1984) and Watmuff et al. (1983) are particularly relevant to the present discussion. These techniques, together with surface-measurement methods, should allow the extension of our knowledge of two- and three-dimensional flows with near separation, separation and/or reattachment. Emphases of this type are necessary and, particularly since experimental activities can require considerable time, should be undertaken soon.

It is also obvious that our understanding of transition is incomplete and available knowledge is such that even the empirical correlation formulas are exceedingly simple. Thus it is not only our knowledge of the mechanisms which is poor but the empirically-based knowledge of the influence of important flow and geometry parameters needs to be strengthened. In this second context can be cited the pressure gradients which give rise to laminar separation bubbles and the extent of the downstream region on which transition

may continue. It does seem, however, that experiments which seek to augment stability such as those of Leipmann et al. (1982) and Straziser and Reshotko (1977) and those which seek to quantify and understand the amplification processes, for example those of Meier, should be increased in number and in effort.

The various approaches to the prediction of transition are all some considerable way from being useful in other than limited ranges of flow. The e^n-method, referred to in Section 2.3, appears to offer the best possibility in the near term, but is in need of much more testing and additional measurements with which to evaluate the tests. These experimental results need not involve the detailed fluid-dynamic processes but, as indicated in the previous paragraph, could be carefully conducted parametric tests which would lead to the improvement and extension of existing correlation formulas as well as allowing the evaluation and development of methods such as e^9.

An alternative approach to the calculation of transition is to make use of turbulence models with low Reynolds-number terms as described by Jones and Launder (1972). The status of this art is, however, very unsatisfactory and little has been achieved since 1972. Indeed, the use of algebraic eddy-viscosity equations such as those of Cebeci and Smith (1974) is vastly to be preferred even though it is necessarily limited. Again, the need for improvement and extension is obvious.

At the same time, it is useful to consider the application of existing turbulence models which include algebraic eddy-viscosity equations, two transport-equation methods such as that which uses the equations for turbulence kinetic energy and dissipation rate and models which solve transport equations for the Reynolds stresses. Procedures based on the simulation of large eddies or on subdivisions of the dissipation equation are not considered since, although they offer promise, they are unlikely to contribute to practical calculations in the near future. Again, reference to the discussion of Cebeci et al. (1984) is appropriate since little has so far been achieved in relation to three-dimensional flows. In their examination of two-dimensional flows, Cebeci et al. formed the view that eddy-viscosity equations were probably appropriate for most aerodynamic flows. This conclusion was based partly on the ability of the different models to represent boundary-layer flows and partly on the relative cost of solving the equations associated with the different models. This conclusion is at best as appropriate to three-dimensional flows where calculation costs are even greater. It is, however, an unacceptable conclusion for the long term since algebraic equations are necessarily limited to the range of experimental information upon which they are based. Turbulence models will always be limited in scope because they are associated with the solution of Reynolds-averaged and, therefore, inexact equations. Undoubtedly, however, greater generality can be achieved with models based on transport equations and basic research is required to discover new formulations and better equations. For the last ten years, advancements to turbulence models have been limited to very ad hoc proposals and to comparison of calculations with measurements which have frequently been rendered of little value by numerical imprecision or extraneous physical effects. New thinking is required and involves the support of new research where a guarantee of an answer cannot always be made.

Wing-Like Geometries

The above recommendations will considerably assist the development of understanding of wing flows and of our ability to calculate their main features. The boundary layer near the junction of a wing and a fuselage, however, needs special attention. Unless special care is taken, the flow is likely to separate ahead of the junction generating an eddy and possibly eddies at the wing-root with a strong horseshoe vortex covering both the upper and lower surfaces of the wing. Apart from the consequent loss of lift and drag gain due to the eddy, the boundary layer on the wing receives an injection of fluid from the fuselage via the eddy boundary with unknown consequences for the flow properties on the wing.

Figure 25 is a sketch of the flowfield at a cross-section of the wing-body region of the airplane. The closed regions marked "eddy" are the two components of the horseshoe vortex; the boundary layer of the fuselage flows over them and may spill over into the wing boundary layer. Figure 25b shows how the horseshoe vortex starts on the fuselage boundary layer as the wing is approached. The adverse pressure gradient on the fuselage ahead of the wing associated with the stagnation at S causes separation ahead of the wing and sets up an eddying region between the oncoming boundary layer and the wing.

A related problem is the flow over a submarine moving under water. A projection from the upper surface known as the sail is a feature of all submarines and the oncoming flow along the hull interacts with the sail in much the same way as at the wing-body junction. The horseshoe eddy generated by the interaction has the additional undesirable feature of being a source of noise which should be reduced as much as possible.

The configuration in Figure 25 is a grossly simplified representation of the wing-body junction region of a large transport aircraft. Indeed many of the modifications of this configuration needed to produce a

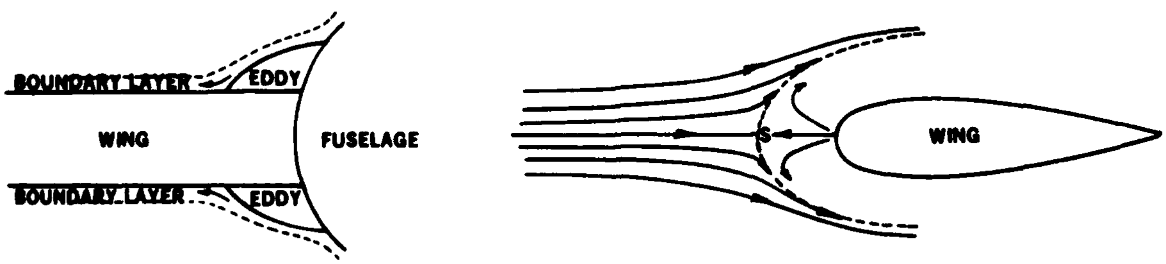

Figure 25. Sketch of the flowfield at a cross section of the wing-body interaction.

modern junction region were introduced to reduce the impact of the separation of the fuselage boundary layer. Included in these modifications are the placing of the wing at the lower part of the fuselage, the introduction of fillets between the wing and the fuselage and the fairing of the fuselage ahead of the wing to smooth the geometrical transition between the fuselage and the wing. Most of these changes have been made as a result of wind-tunnel observations and theoretical investigations have played only a small role in these improvements to the airplane's flight characteristics.

The wing-body junction is one example of a class of flow problems sometimes referred to as corner flows. Others include wing pylon intersections and intersecting tail surfaces. Since the wing-body flow generates initial conditions for the boundary-layer calculations on the wing, it is important to be able to calculate the flow properties in this region. Because of the presence of strong secondary flows, boundary-layer ideas may not be adequate to treat all such flows. However, for configurations such as blended wing-fuselage combinations or extensive wing-fuselage filleting which reduce the impact of the secondary flows and limit the size of regions of recirculating flow, the boundary-layer approach can be sufficient. Progress in such flows or in those with separation, however, has not been satisfactory. There are two reasons for the lack of fundamental research in this important area. First, there are few experiments to help form a true understanding of the nature of the interaction. One of these is by East and Hoxey (1969) who investigated the advent of separation on the fuselage and Fig. 25 is obtained from their paper. Another experiment on the subject is due to Pierce and McAllister (1982). Neither contribution discussed the influence of their secondary flow on the flow over the wing. The second reason is that an efficient method of computing the three-dimensional boundary layer has only recently become available.

One approach to the calculation of corner flows is to make use of a parabolical form of the Navier-Stokes equations in which the flow normal to the direction of main flow is represented by a two-dimensional form of the Navier-Stokes equations, with laminar and turbulent diffusion correctly represented in two orthogonal directions, and with marching in the main-flow direction. This type of approach has been used, for example by Pratap and Spalding (1976) to represent three-dimensional duct flows. Kreskovsky, Briley and McDonald (1981) have made use of a combination of potential-flow and Navier-Stokes equations and have been even more successful in their attempts to represent bend flows of rectangular cross section and the corner flows which they embody. Inviscid approaches to represent the flows in the blade passages of turbines have also been reported, for example Denton (1982), and have been remarkably successful. This suggests that the turbulence model can be simple since the phenomena tends to be pressure controlled.

There remains, however, the representation of the leading-edge region at which the vortex is formed. This appears to be a complex phenomenon which may require the solution of the full Navier-Stokes equations. Experimental information is difficult to obtain in this region and new attempts are required. They should encompass a range of geometrical arrangements, including a large variation in the thickness of the fuselage boundary layer, and be performed with detail sufficient to describe the leading-edge flow so that calculation methods could start from a location downstream of the leading edge.

Apart from this particular region of flow, measurements are required for a wider range of wing flows which should include cruise and takeoff Mach numbers and slot-flap configurations. The available information is limited and needs to be considerably expanded and improved.

Missile-Like Geometries

In recent years, considerable progress has been made in the computation of the inviscid flows around bodies of revolution at high speeds, including the effect of added wings and fins such as occur on missile configurations. A particularly effective contribution has been made by Wardlaw et al. (1981) using a method based on the Euler equations. The agreement achieved between experiment and theory, and the generally plausible flow features when detailed comparisons are not available, are impressive, given the present state of knowledge. In particular, the circulation of the secondary flow behind the separation line and on the leeside of the fin when it has a subsonic leading edge, are encouraging results from this and other papers concerned with the solution of Euler equations. The most extensive quantitative comparisons between experiment and theory concern the pressure distribution on the fins when they have supersonic leading edges. On the whole, the agreement is considered to be satisfactory, the exceptions being the neighborhood of the leading-edge where the theory underpredicts strong shocks, at the fin-body junction and near the trailing edges in certain configurations, see Figure 26, for example. When the leading edge is transonic and the associated shock is not attached, the comparisons are less detailed but still encouraging; the only significant discrepancies being in the pressure distribution on the body at stations towards the trailing edge of the fin. The agreement between measured and calculated wing pressures is good. Finally subsonic leading edges are considered; detailed comparisons are confined to the normal force coefficient and center-of-pressure and are also favorable. Wardlaw et al. (1981) suggested that the discrepancies are due in part to boundary-layer effects and, of course, it is generally believed that separation is intimately connected with the boundary layer. There is, as a consequence, a prime need to develop, in association with the inviscid code, an interactive boundary-layer code which will reduce the discrepancies in pressure already mentioned, at the fin trailing edge, on the body and at the fin-body junction. For example, consider the trailing-edge region of the fins. Here the boundary layer is relatively thick and the equivalent blowing velocity, v_e, which may be computed by standard boundary-layer theory, may be sufficiently large to cause significant modifications to the pressure distribution. An associated area is the region of the body near the trailing edge of the wings where discrepancies in the pressure distribution have been found when the wings have transonic trailing edges. A calculation of v_e in this region is likely to lead to improved estimates of the pressure.

The fin-body junction is a more difficult region to discuss properly and may need an interactive approach since the direct boundary-layer method can only carry the calculation as far as separation, which possibly occurs on the body ahead of the fin leading-edge. It certainly does if the leading edge is rounded at the junction; further work is necessary to determine whether a sharp leading edge makes a significant difference and, where appropriate, to develop an interactive theory to enable us to advance the solution up to the fin. It appears that sufficient knowledge is now available to allow the formulation of a coordinated research program to tackle many of the remaining problems in missiles. This should undoubtedly include

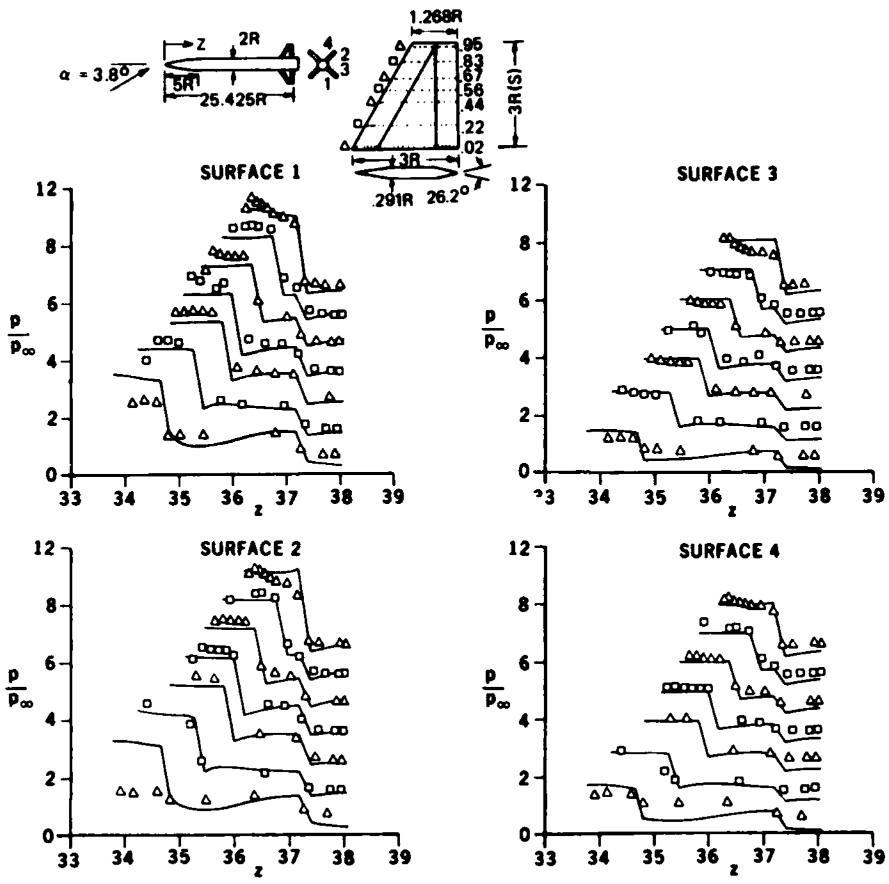

Figure 26. Comparison of calculated and measured pressure distributions on the four surfaces.

approaches to the solution of the Navier-Stokes equations. It is to be expected, however, that more reliable results will be obtained in the short and medium term from the development and use of interactive methods.

The experiments proposed in relation to bodies of revolution and wing-fuselage intersections will assist the development of calculation methods for missiles. In this case, however, supersonic flow and sharp leading edges have increased the range of variables for which information is required. Many requirements are similar to those of fighter airplanes, even more so if air-breathing engines are used to power the missile, and add emphasis for the required development.

Ship-Like Geometries

The proposed research on interactive methods will assist the calculation of the flow around the hull of ships and particularly in the region close to the stern separation. Problems associated with bulbous bows remain and will be assisted by the proposed work on bodies of revolution, but further research is essential to deal with the region of rapid curvature change immediately behind the bow and at the stern. Improved interactive methods will allow those flows to be better represented and, in addition, the new parabolized Navier-Stokes procedure presently being developed at the David Taylor Naval Ship Research and Development Center deserves special support since it is likely to be required to represent the flows, where the curvature causes secondary flows. Even with the solution of the parabolized Navier-Stokes equations, a three-dimensional viscous-inviscid procedure will be required to determine initial and outer boundary conditions.

As indicated earlier, there is little experimental data to support the development of calculation methods for ship hulls and additional work is clearly necessary, particularly to determine the characteristics of the flow in the region of rapid curvature change, in the near separation and separated flow regions and in the outer region of the thick boundary layers which occur towards the stern.

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SUMMARY REMARKS

by

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The purpose of the round table discussion, to review recent progress and to provide future guidance for activities on the subject of three-dimensional boundary layers, was successfully achieved. The discussions by the presenters, interactive discussions among the attendees, and the additional information in the expanded written versions of the presentations provide a comprehensive assessment of the current status and needed developments in theory and experiments. Those interested in the subject should review the contents of the papers in this report carefully and utilize the extensive reference lists for additional information. However, it is probably of some value to briefly indicate some areas that need consideration in future research and application. These requirements, although not a complete list, appear repeatedly in discussions of many different efforts in different countries.

Turbulence modelling, which is already unsatisfactory in two-dimensional flows, is even less suitable for three-dimensional boundary layers. Although there are some good comparisons with experiment for attached flows, there is no consistency for a particular model for different applications. More sophisticated models do not appear to work better than simple models for many cases. The situation is even less satisfactory for separated flows where the validity of the computations themselves cloud the issue.

Flow separation in general exposes deficiencies in treating viscous/inviscid interactions which, although always important, are crititical when separation is present. Proper viscous/inviscid coupling must be seriously addressed for almost any realistic three-dimensional boundary-layer flow. Some computations using inverse techniques show promise for separated flow.

Lack of an accurate, extensive experimental data base hampers effort to gain insight into important flow mechanisms and to validate computational results. The extensive review in these proceedings is striking in the relatively limited amount of high quality experimental data cited. Any future research thrust on this subject should include well-planned and carefully executed experiments to provide a data base that will challenge the development of computational methods and provide basic insight into important viscous phenomena.

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14. Abstract

This Report contains papers presented by the ~~contributors~~to an AGARD Fluid Dynamics Panel Round Table Discussion held in ~~Brussels, Belgium,~~ ~~in~~May 1984. The Discussion was part of the Panel's Symposium devoted to Improvement of Aerodynamic Performance through Boundary Layer Control and High Lift Systems, reported in AGARD Conference Proceedings CP 365.

The purpose of the Discussion was to review recent progress in, and to provide guidance for future work in the field of 3-D boundary layers. Turbulence modelling, which is unsatisfactory in 2-D flows, is even less suitable for three-dimensional boundary layers. More sophisticated models do not appear to work better in many cases than simple ones. The situation is even less satisfactory for separated flows where the validity of certain computations is in question.

Lack of accurate, extensive data hampers efforts to comprehend important flow mechanisms and validate computational results. The extensive review reported at the Conference and in this Discussion is remarkable in the paucity of high-quality experimental data cited.

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